Phase Decomposition for Loudspeaker Analysis

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Abstract: The sound pressure level from a loudspeaker at a distance is determined mainly by the vibration of the cone(s). It is however difficult to make a direct correlation between the displacement on the radiating surface(s) and the resulting sound pressure. In order to establish insight into this correlation some kind of decomposition of the vibration is advantageous. The so-called phase decomposition described by Klippel [1] splits the displacement into an in-phase, an anti-phase and a quadrature component, with each component contributing differently to the pressure. The decomposition requires that the pressure be calculated via the Rayleigh integral. Klippel provides software which can perform the analysis for a measured displacement pattern, but it was desired to have the phase decomposition done within COMSOL Multiphysics. The Rayleigh integral and the phase decomposition technique were successfully implemented, and have been applied to vibrating cones and vibrating cabinets.

Keywords: Loudspeaker, cone, Rayleigh integral, phase decomposition

1. Introduction

Vibroacoustic simulations of loudspeakers enable engineers to make virtual prototypes and examine characteristics such as frequency response, impedance and spatial radiation patterns. The simulations have geometric and material representations of the drivers and enclosures, and with proper knowledge of the total assembly, such simulations can provide valuable insight in advance of actually having a prototype. Additionally, the simulations offer information that is either difficult or impossible to measure in practice.

At times though, having the results from a standard simulation is not enough to provide a satisfactory correlation between structural vibrations and the resulting sound pressure. For example, a dip in the acoustic frequency response may be a result of the vibration of the cones being so that certain parts of the cones have different phases than other parts, thus causing interference. For complex vibrations it is often difficult for the engineer to assess the phase components across a cone. However, with structural and acoustic decomposition techniques, the displacement of the cone can be decomposed into components that add to the sound pressure, subtract from it, or do not contribute to the sound pressure at all.

Certain assumptions have to be made when constructing a sensible phase decomposition. In the present work it is assumed that the vibrating surface is flat and that this surface is situated in infinitely large and flat baffle. It is also assumed that there are no obstructions in the pathway between the surface and the observation point. With these assumptions only a single displacement direction needs to be considered and there are no diffraction contributions.

Even though the phase decomposition is only exact for plane surfaces placed in an infinite baffle, the error made by not meeting these requirements is small enough for many applications, that the technique is still useful for gaining knowledge about the application and subsequently making engineering decisions.

2. Theory

Consider the situation depicted in Figure 1. A flat surface with a certain vibrational characteristics is located in a flat and infinite baffle. At each point on the surface a complex displacement is known. The sound pressure is to be calculated in an observation point.

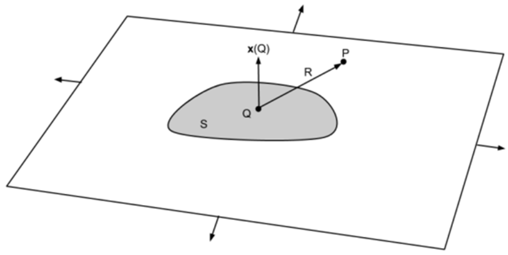

Figure 1. A flat surface $\boldsymbol{S}$ is placed in an infinite baffle. At point $\boldsymbol{Q}$ the total surface displacement is $\boldsymbol{w}(\boldsymbol{Q})$. The distance from point $\boldsymbol{Q}$ to an observation point $\boldsymbol{P}$ is $\boldsymbol{R}$.

An implicit time dependency of $e^{i\omega t}$ is assumed throughout, so that only the phasors of the variables are considered in the following.

For the flat surface S depicted in Figure 1 the general total displacement $\vec{\boldsymbol{u}} = (\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w})$ only has one spatial phasor component, i.e. $\vec{\boldsymbol{u}} = (\boldsymbol{0}, \boldsymbol{0}, \boldsymbol{w})$. The total complex sound pressure $\boldsymbol{p}(P)$ in observation point P resulting from the complex displacement $\boldsymbol{w}$ on the surface S can be calculated via the Rayleigh integral [2]

$$\boldsymbol{p}(P) = \frac{-\omega^2 \rho}{2\pi} \int_S \boldsymbol{w}(Q) \frac{e^{-ikR}}{R} dS.$$

Here, ρ is the density of the acoustic medium, ω is the angular frequency, and the wavenumber $k = \frac{\omega}{c}$, where c is the sound speed.

The complex displacement $\boldsymbol{w}$ on the flat vibrating surface can be decomposed into three complex components [1]

$$\boldsymbol{w} = \boldsymbol{w}_{in} + \boldsymbol{w}_{anti} + \boldsymbol{w}_{quad}$$

relative to the observation point P. Each component is also frequency-dependent. The in-phase component $\boldsymbol{w}_{in}$ adds positively to the total pressure at the observation point, the anti-phase component $\boldsymbol{w}_{anti}$ subtracts from the pressure and the out-of-phase (or quadrature) component $\boldsymbol{w}_{quad}$ does not contribute to the pressure.

The individual components are calculated as[1] [1]

$$\boldsymbol{w}_{in}(Q, \boldsymbol{p}, R) = Re^{+}\left(\boldsymbol{w}(Q)\, e^{-i\varphi_{ref}}\right) e^{i\varphi_{ref}}$$
$$\boldsymbol{w}_{anti}(Q, \boldsymbol{p}, R) = Re^{-}\left(\boldsymbol{w}(Q)\, e^{-i\varphi_{ref}}\right) e^{i\varphi_{ref}}$$
$$\boldsymbol{w}_{quad}(Q, \boldsymbol{p}, R) = Im\left(\boldsymbol{w}(Q)\, e^{-i\varphi_{ref}}\right) e^{i\left(\varphi_{ref} + \frac{\pi}{2}\right)}$$

where

$$\varphi_{ref} = arg\left(\boldsymbol{p}(P)\right) + \pi + kR.$$

The reasoning behind the above expressions is explained in the Appendix. From the displacement components the respective sound pressure components can be calculated via the Rayleigh integral.

For a non-flat surface S the Rayleigh integral is inaccurate, regardless of the displacement component being input. However, when calculated anyway, the displacement to be input is chosen here as the spatial component of the total displacement in the direction of the baffle normal, i.e. $\boldsymbol{w}$. This complies with a laser scanning for which Klippel's scanning software is intended. Since the total displacement $\vec{\boldsymbol{u}}$ will have more than one non-zero spatial component for a non-flat surface, the individual phase decomposed displacement components will not sum exactly to the total displacement. However, for applications like shallow loudspeaker the phase decomposition will still provide insight to the relation between the displacement and the pressure.

3. Use of COMSOL Multiphysics

The Rayleigh integral and the phase decomposition functionality were implemented using the integration operator and general mathematical functions readily available in COMSOL Multiphysics.

Both a 3D version and a 2D-axisymmetry version of the phase decomposition have been implemented. The sound pressure calculated via the Rayleigh integral was validated via a comparison with an *Acoustic-Solid Interaction, Frequency Domain* simulation. The phase decomposition functionality was subsequently tested, e.g. by applying designed displacements with certain phase characteristics, and examining the corresponding sound pressures.

[1] Be aware that some available literature contains erroneous or incomplete expressions.

A 3D loudspeaker application will typically be examined using solely a *Solid Mechanics* Physics Interface simulation to avoid having to mesh a fluid medium. Additionally, when a loudspeaker driver is examined, its electromagnetic properties may be included via the *Electric Circuit* Physics Interface found in the *AC/DC* Module.

4. Applications

The decomposition functionality is advantageous in loudspeaker development, illustrated here with a generic case and a small full-range loudspeaker driver.

1.1 Illustration case

Consider a flat disc with radius a of 0.1 m in an infinite baffle. On its surface the disk has a total force applied in the direction of the baffle normal of

$$F_z = e^{-\frac{2}{a}\sqrt{\left(x-\frac{a}{5}\right)^2+\left(y-\frac{a}{5}\right)^2}}\,N.$$

The resulting distinct displacement pattern can be used for illustrating the phase decomposition technique.

In Figure 2 the total displacement is shown for a frequency of 1.6 kHz. Two distinct displacement bulges in opposite phase are seen.

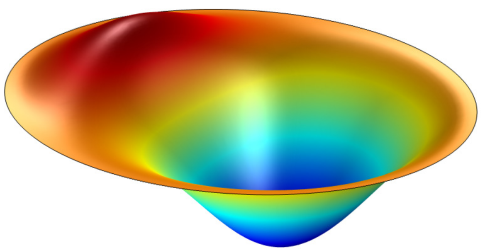

Figure 2. Total displacement of the vibrating disc at 1.6 kHz.

The displacement was phase decomposed relative to an observation point on-axis in a distance of 1 m above the disk. All displacement components are depicted using the same amplitude scale as in Figure 2. The in-phase component is shown in Figure 3. It can be seen that the in-phase part is simply the larger of the two displacement bulges.

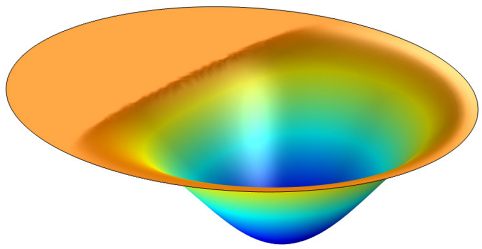

Figure 3. In-phase displacement component of the vibrating disc at 1.6 kHz.

The anti-phase shown in Figure 4 is the smaller of the displacement bulges. Since the total displacement only has two phase values, this has to be case, since the anti-phase component can never be larger than the in-phase component.

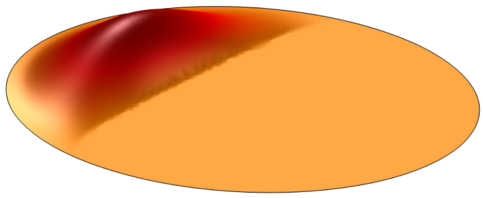

Figure 4. Anti-phase displacement component of the vibrating disc at 1.6 kHz.

Finally, the out-of-phase displacement is shown in Figure 5. This component is essential zero. For the total displacement pattern shown in Figure 2, the out-of-phase component can only be non-zero on-axis if the observation point is located close to the surface.

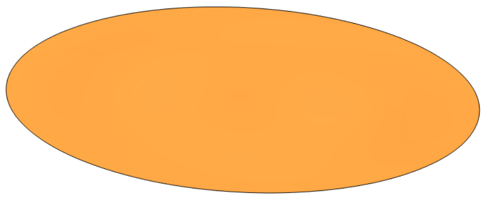

Figure 5. Out-of-phase displacement component of the vibrating disc at 1.6 kHz.

The individual sound pressure level components are shown in Figure 6. It is seen that below 1 kHz the displacement is entirely in-phase, whereas above 1 kHz the total sound pressure level is composed of the in-phase and anti-phase parts.

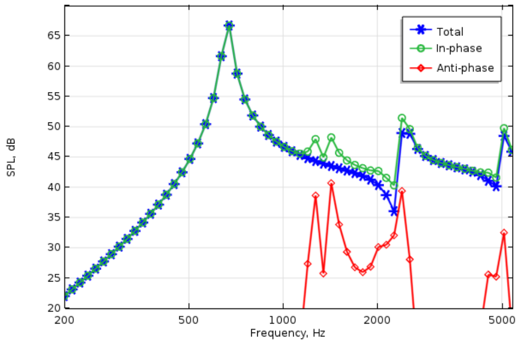

Figure 6. The total, in-phase and anti-phase sound pressure level components for the vibrating disk, respectively, relative to an observation point located on-axis 1 meter above the disc.

1.2 Cone vibration analysis

A 3 inch full-range speaker was examined. Its operating range is approximately 200 Hz to 10 kHz, and it is intended for use in a car. The driver is shown is Figure 7.

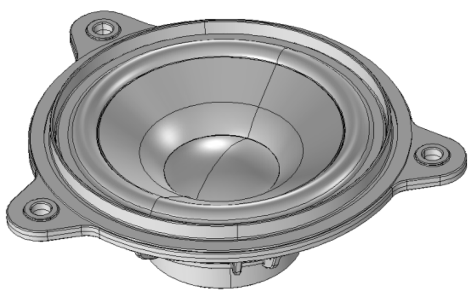

Figure 7. The 3 inch driver subjected to phase decomposition analysis.

The driver has a high degree of axial symmetry apart from its wiring, and so a 2D-axisymmetry *Acoustic-Solid Interaction, Frequency Domain* study including an *Electric Circuit* Physics Interface containing the electromagnetics characteristics of the driver, was carried out, and the phase decomposition functionality was applied.

The sound pressure levels for the different phase component can be seen in Figure 8. At frequencies above 3.5 kHz the anti-phase component influences the total response.

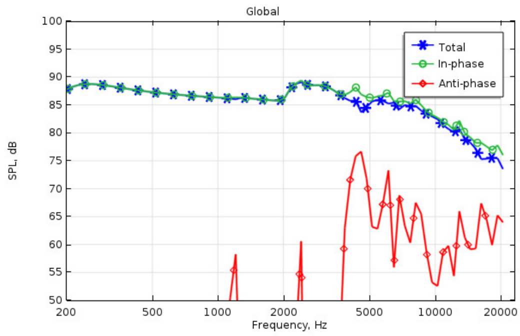

Figure 8. The total, in-phase and anti-phase sound pressure level components for the 3 inch driver are shown as solid, dotted and dashed lines, respectively.

The displacement on the driver surface at 4.5 kHz was examined and decomposed for an observation point *P* on-axis at a distance of 1 m away from the baffle surface.

The in-phase, anti-phase and out-of-phase displacements[2] for the chosen frequency and observation point are shown in Figure 9, Figure 10, Figure 11 and Figure 12, respectively. All displacement components are depicted using the same amplitude scale as in Figure 9. It should be noted that since the total displacement depicted has two spatial (and complex) phasor components, i.e. $\boldsymbol{u}$ and $\boldsymbol{w}$ (with their real parts shown), but the in-phase, anti-phase and out-of-phase components only have one spatial component each, the components shown do not sum exactly to the total displacement shown. This is a consequence of the surface being non-flat.

[2] All displacements shown are shifted $\frac{\pi}{4}$ radians relative to an input voltage of 2.83+0i V, to have an optimal view of the vibration patterns.

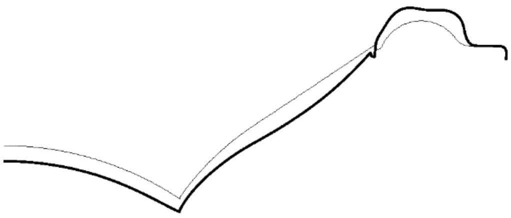

Figure 9. The total displacement at 4.5 kHz for the driver is shown.

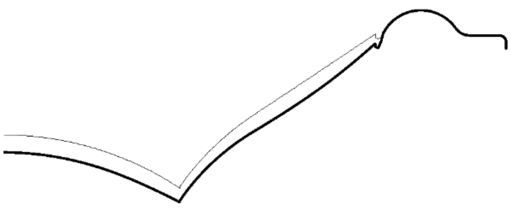

Figure 10. The in-phase displacement component at 4.5 kHz involves only the cone, whereas the surround has no in-phase displacement.

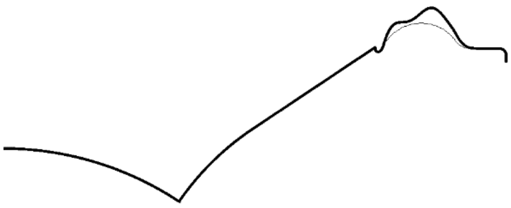

Figure 11. The anti-phase displacement component at 4.5 kHz involves only the surround.

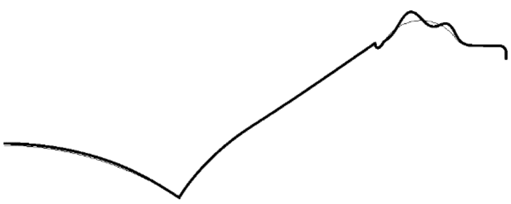

Figure 12. The out-of-phase displacement component at 4.5 kHz involves only the surround.

It can be seen from the analysis that the dip at 4.5 kHz in the total sound pressure level is due to a peak in the anti-phase component originating from the movement of the surround; the softer part connecting the cone to the basket. The surround also has a significant amount of out-of-phase displacement at this frequency. The knowledge gained about how the different parts of the radiating surface contribute to the pressure on-axis, will aid in optimizing the topology and/or material parameters.

2. Conclusion

A phase decomposition technique has been implemented in COMSOL Multiphysics. The phase decomposition allows the user to relate the sound pressure in an observation point to several displacement components on a vibrating surface. An in-phase component adds positively to the pressure, an anti-phase component subtracts from the pressure and an out-of-phase component neither adds to nor subtracts from the total pressure. The three components add up to the total displacement on a flat surface.

For non-flat surfaces the displacement components do not sum exactly to the total displacement. However, the phase decomposition still reveals the characteristics of the displacement, i.e. if a dip in the sound pressure level is simply due to lower total displacement at that frequency, or if the anti-phase component interferes more at that frequency.

The phase decomposition was applied to practical cases, demonstrating e.g. how the loudspeaker surround can produce significant anti-phase and out-of-phase displacement components which heavily affect the sound pressure.

3. References

1. W. Klippel and Joachim Schlecter, Measurement and Vizualization of Loudspeaker Cone Vibration, www.klippel.de
2. M. Bruneau and T. Scelo, *Fundamentals of Acoustics*, pp 327. ISTE Ltd, 2006

4. Acknowledgements

The author would like to express his warm thanks to Mads Jakob Herring Jensen, Technical Product Manager, Acoustics at COMSOL A/S, for helping with the implementation of the phase decomposition.

5. Appendix

This appendix explains how the transformation from the (real, imaginary) orthonormal displacement basis to the (phase/anti-phase, out-of-phase) orthonormal displacement basis is carried out.

For a given point on the flat vibrating surface in Figure 1 the complex displacement can be decomposed with respect to the pressure in a given observation point. The phase $arg\big(\boldsymbol{p}(P)\big)$ of the complex pressure in the observation point will be lagging the phase $arg\big(\boldsymbol{p}(P)\big)$ of the displacement on the surface point. If the phase lag is exactly equal to the phase difference corresponding to the distance travelled by the acoustic wave at the given frequency (and adding an extra phase of π radians to account for the sign in equation [1]), then the displacement is entirely in-phase with the pressure. This is illustrated in Figure 13. Similarly, the out-of-phase component is in quadrature with the in-phase component, and the anti-phase component is offset π radians to the in-phase component.

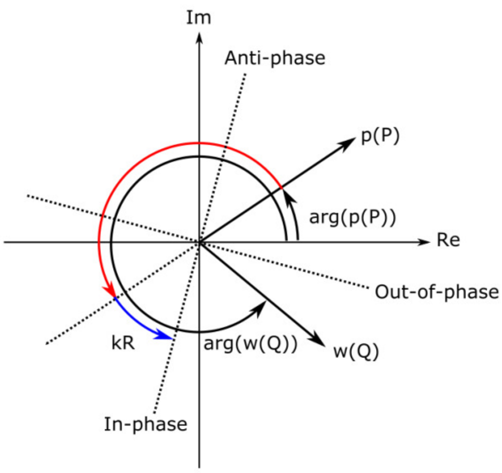

Figure 13. An overview of the phasor components involved in the decomposition technique.

Expressions for the individual components are most easily found by rotating all phasors backwards (clockwise) by a reference phase

$$\varphi_{ref} = arg\big(\boldsymbol{p}(P)\big) + \pi + kR.$$

By multiplying all phasors by $e^{-i\varphi_{ref}}$, the in-phase component is aligned with the real-axis. So the in-phase displacement is found as the projection of the total displacement onto the positive part of the real axis. Using a Real-operator on the (rotated) total displacement and taking its positive part gives the amplitude of the in-phase component, and by subsequently rotating this phasor back to its origin by multiplying by $e^{i\varphi_{ref}}$, the in-phase component is found as

$$\boldsymbol{w}_{in}(Q,\boldsymbol{p},R) = Re^{+}\big(\boldsymbol{w}(Q)e^{-i\varphi_{ref}}\big)e^{i\varphi_{ref}}.$$

Similarly, the anti-phase displacement component is found as the negative part of the projection onto the real-axis

$$\boldsymbol{w}_{anti}(Q,\boldsymbol{p},R) = Re^{-}\big(\boldsymbol{w}(Q)e^{-i\varphi_{ref}}\big)e^{i\varphi_{ref}}.$$

Finally, the out-of-phase displacement component is found as the projection onto the imaginary axis, with a rotation back to its origin taking into account that the out-of-phase displacement is a quadrature component

$$\boldsymbol{w}_{quad}(Q,\boldsymbol{p},R) = Im\big(\boldsymbol{w}(Q)e^{-i\varphi_{ref}}\big)e^{i\left(\varphi_{ref}+\frac{\pi}{2}\right)}.$$