COMSOL CONFERENCE
2017 ROTTERDAM

Topology Optimization of Thermoviscous Acoustics in Tubes and Slits with Hearing Aid Applications

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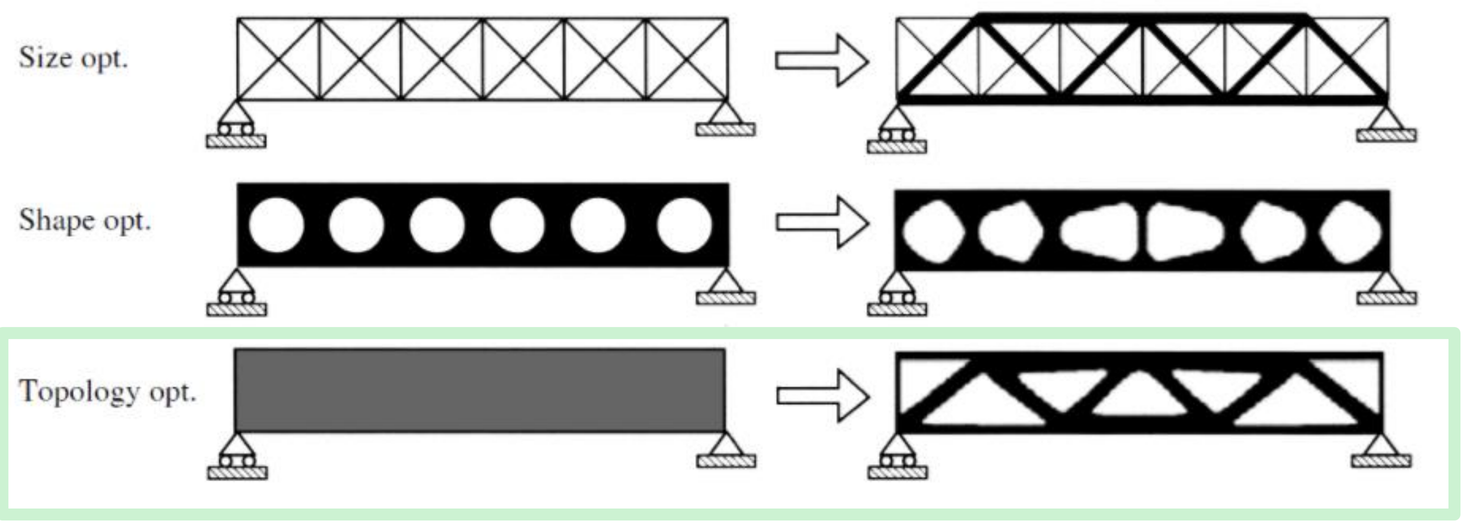
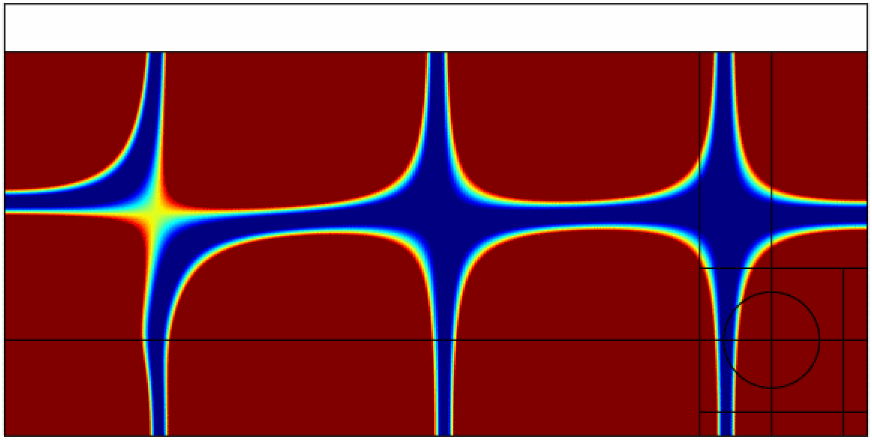
- Topology optimization has its origin in structural mechanics:



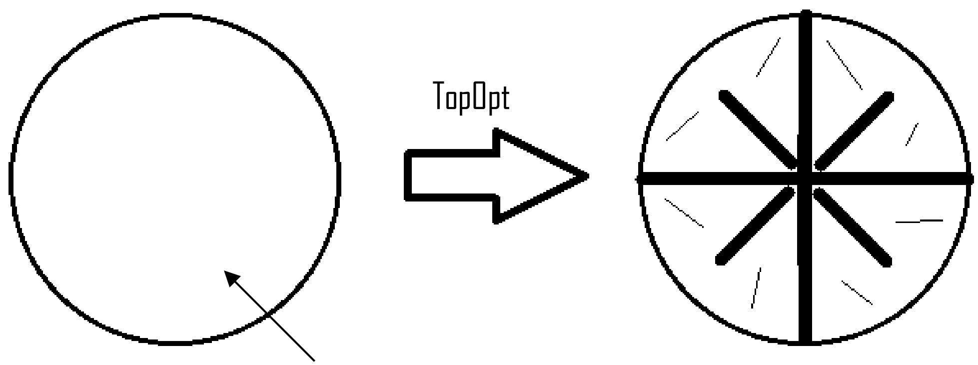
"Topology Optimization – Theory, Methods And Applications", M.P. Bendsøe and O. Sigmund, Springer

- Acoustic topology optimization has also been investigated in recent years:



"Acoustic design by topology optimization", Düring et al., JSV 317 (2008) pp 557-575
"How to Use Acoustic Topology Optimization in Your Simulations Studies", René Christensen, COMSOL blog

- We wish to establish a method for doing topology optimization in acoustic tubing systems with thermoviscous effects included



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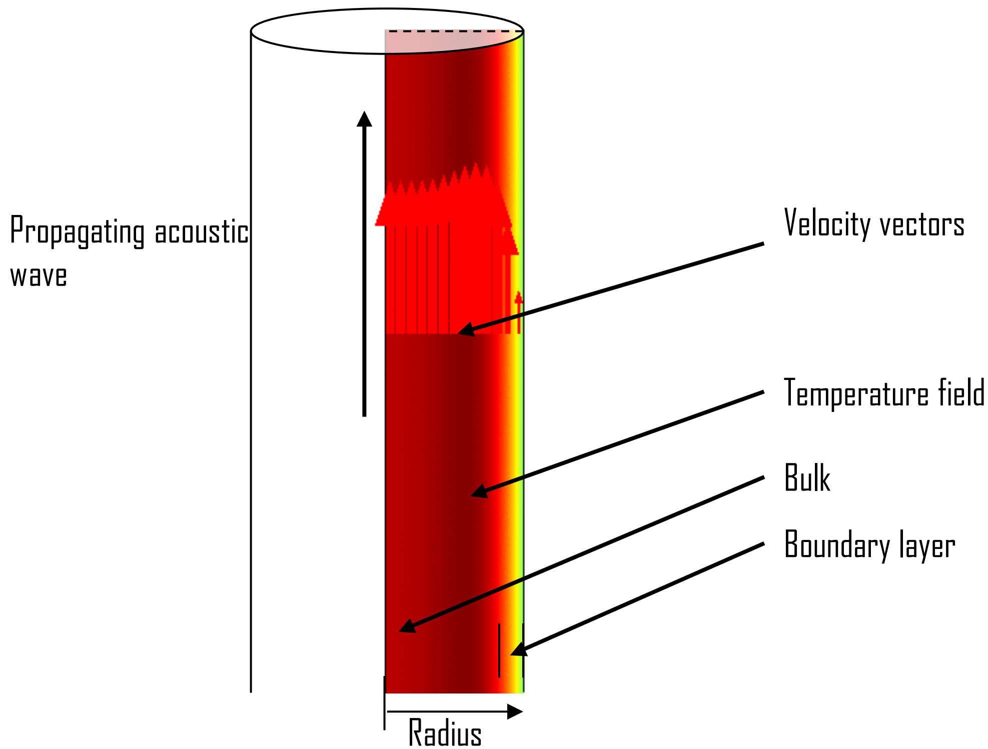
- Thermoviscous acoustics in small geometries



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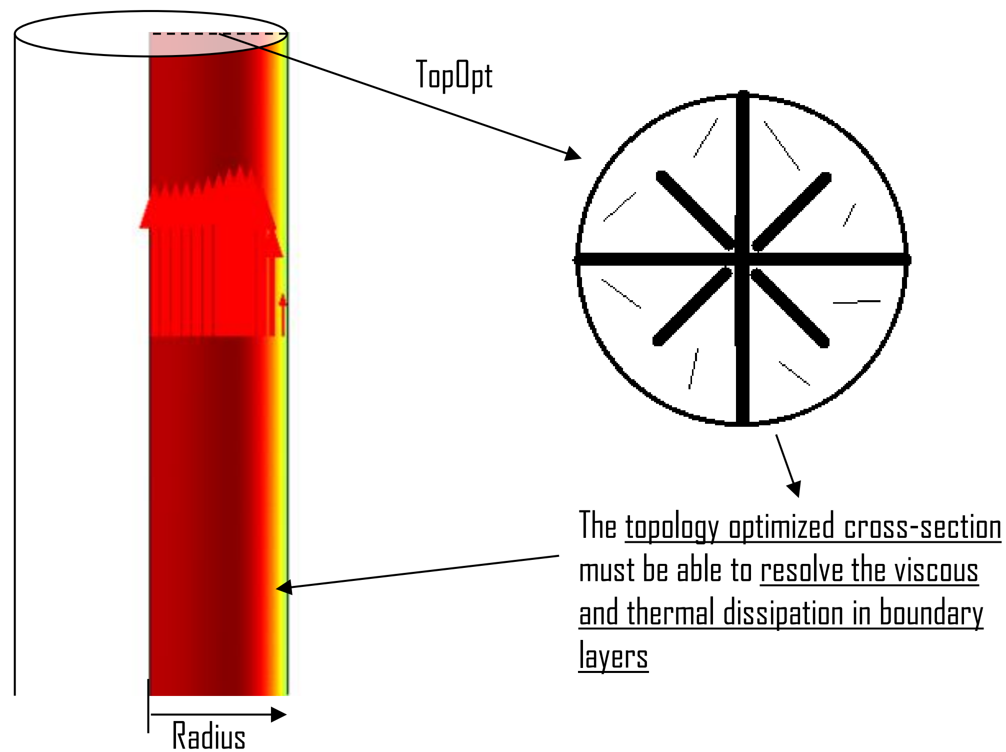
- Thermoviscous acoustics topology optimization



- Full Linearized Navier-Stokes Model

$$i\omega\rho_0\vec{v} - \left(\frac{4}{3}\mu + \eta\right)\nabla\cdot(\nabla\cdot\vec{v}) + \mu\nabla\times(\nabla\times\vec{v}) + \nabla p = 0$$

$$i\omega\rho_0 C_p T - \kappa\Delta T - i\omega p = 0$$

$$\nabla\cdot\vec{v} - i\omega\frac{T}{T_0} + i\omega\frac{p}{p_0} = 0$$

- Low Reduced Frequency Model

$$\Psi_v + k_v^{-2}\Delta_{cd}\Psi_v = 1$$

$$\Psi_h + k_h^{-2}\Delta_{cd}\Psi_h = 1$$

$$\rho = \frac{\rho_0}{\frac{1}{\Omega_{cd}}\int_{\Omega_{cd}}\Psi_v d\Omega_{cd}}$$

$$K = \frac{K_0}{\gamma - (\gamma - 1)\frac{1}{\Omega_{cd}}\int_{\Omega_{cd}}\Psi_h d\Omega_{cd}}$$

$$\Delta_{pd}p + k_0^2 \frac{\gamma - (\gamma - 1)\frac{1}{\Omega_{cd}}\int_{\Omega_{cd}}\Psi_h d\Omega_{cd}}{\frac{1}{\Omega_{cd}}\int_{\Omega_{cd}}\Psi_v d\Omega_{cd}} p = 0$$

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- Low Reduced Frequency Model

$$\Psi_v + k_v^{-2}\Delta_{cd}\Psi_v = 1$$

$$\Psi_h + k_h^{-2}\Delta_{cd}\Psi_h = 1$$

$\longrightarrow$

$$\rho = \frac{\rho_0}{\frac{1}{\Omega_{cd}}\int_{\Omega_{cd}}\Psi_v d\Omega_{cd}}$$

$$K = \frac{K_0}{\gamma - (\gamma - 1)\frac{1}{\Omega_{cd}}\int_{\Omega_{cd}}\Psi_h d\Omega_{cd}}$$

$\updownarrow$

Transmission line parameters:

$$Z' \equiv R' + i\omega L' = i\omega\rho/\Omega_{cd}$$

$$Y' \equiv G' + i\omega C' = i\omega\Omega_{cd}/K$$

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- Topology optimization scheme

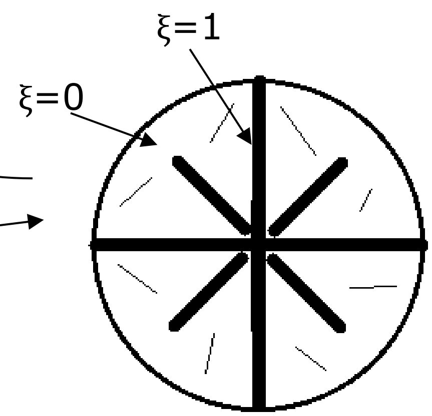
$$\Psi_v + k_v^{-2}\Delta_{cd}\Psi_v = 1$$

$$\Psi_h + k_h^{-2}\Delta_{cd}\Psi_h = 1$$

$$\min_{\xi}: \Phi(\xi, \Psi_v, \Psi_h)$$

$$\text{s.t.}\begin{cases} 0 < \xi(x,y) \leq 1 \quad \forall\,(x,y) \in \Omega_{cd} \\ \int_{\Omega_{cd}} \xi d\Omega_{cd} \leq \beta\Omega_{cd} \end{cases}$$

?



- <u>Topology optimization scheme</u>

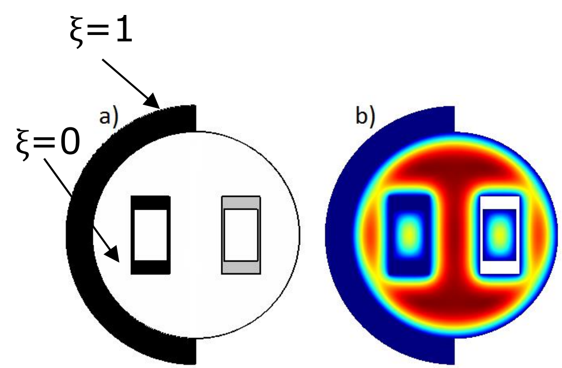
$$a_v(\xi)\Psi_v + k_v^{-2}\Delta_{cd}\Psi_v = f_v(\xi)$$

$$a_h(\xi)\Psi_h + k_h^{-2}\Delta_{cd}\Psi_h = f_h(\xi)$$

$a_v(\xi)$ and $f_v(\xi)$ are interpolated via e.g. SIMP or RAMP

$$\min_{\xi}: \Phi(\xi, \Psi_v, \Psi_h)$$

$$\text{s.t.} \begin{Bmatrix} 0 < \xi(x,y) \leq 1 \quad \forall\, (x,y) \in \Omega_{cd} \\ \int_{\Omega_{cd}} \xi d\Omega_{cd} \leq \beta\Omega_{cd} \end{Bmatrix}$$



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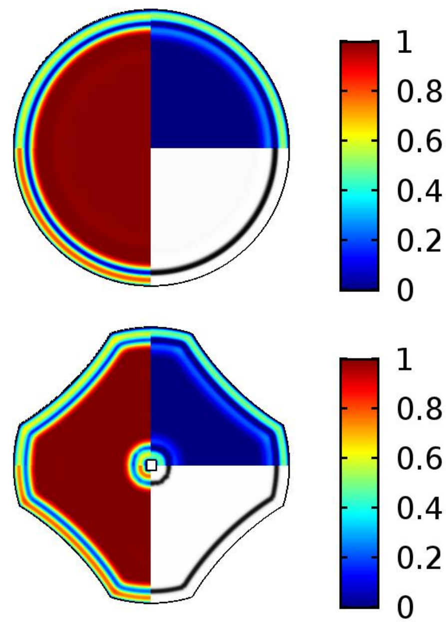
- Hearing aids tube

$$\max_{\xi}: \Phi = \Re(R') = \omega\rho_0\Re\left(\frac{i}{\int_{\Omega_{cd}(\xi)} \boldsymbol{\Psi}_v d\Omega_{cd}}\right)$$

$$\text{s.t.}: \begin{cases} \mathbf{0 < \boldsymbol{\xi} \leq 1} \\ \int_{\Omega_{cd}} \boldsymbol{\xi} d\Omega_{cd} / \Omega_{cd} \leq \beta \\ \mathbf{S}_{\boldsymbol{\phi}} \boldsymbol{\Psi}_{\boldsymbol{\phi}} = \mathbf{f}_{\boldsymbol{\phi}} \end{cases}$$



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- An <u>acoustic topology optimization</u> scheme was proposed which includes <u>thermal and viscous dissipation</u>
- The optimization scheme assumes <u>constant cross-section geometries</u>
- The scheme was <u>implemented entirely in COMSOL Multiphysics</u> using the *Optimization* and the *PDE* interfaces, both found in the *Mathematics* module
- The technique provides insight into finding <u>optimized geometries</u> for different vibro-acoustic objective functions, with <u>patents pending</u>

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