

The Virtual Aquarium: Simulations of Fish Swimming

M. Curatolo and L. Teresi*

LaMS – Modelling & Simulation Lab,
Dept. Mathematics & Physics, Università Roma Tre, Italy

* Corresponding author: Via Vasca Navale 84, 00146, Roma, Italy, email: teresi@uniroma3.it

Abstract: Our goal is to reproduce the key features of carangiform swimming by running 2D simulations which fully exploit the Fluid-Structure Interaction interface of COMSOL. Fish swimming is an important area of research, with relevant developments on biomechanics, robotics and mathematical modeling. Usually, in fish swimming simulations, the motion of the fish is assigned, and much efforts are put on fluid dynamics. Here, we simulate muscles contraction by using the notion of distortions (also known as pre-strains), emphasizing the kinematical role of muscle, the generation of movement, rather than the dynamical one, the production of force. A proper undulatory movement of a fish-like body is reproduced by defining a pattern of muscles activation, tuned both in space and in time.

Keywords: Fish swimming, Fluid-Structure Interaction, Muscle Modeling.

1. Introduction

We present a virtual aquarium where it is possible to simulate fish swimming by exploiting the Fluid-Structure Interaction (FSI) interface of COMSOL. Fish swimming is an important area of research with relevant developments on biomechanics, robotics and mathematical issues.

Two-dimensional analyses of fish locomotion have shown that even fishes of very different body types such as eels, trout, mackerel, and tuna show extremely similar patterns of body movement when viewed in a horizontal section during steady undulatory locomotion [1].

Usually, in fish swimming simulations, the motion of the fish is assigned, and much efforts are put on fluid dynamics [2]. Here, following [3], we simulate muscles contraction by using the notion of distortions (also known as pre-strains), emphasizing the kinematical role of muscle, the generation of movement, rather than the dynamical one, the production of force; within this framework,

force arises in muscles only when motion is hampered by some sort of constraints. In our case, the lateral motion of the fish is reacted upon by the force exerted by the surrounding fluid. The muscle actions are modeled by a time-evolving distortion field that produces the sought flexural motion of the fish-like body; using a few key parameters, it is possible to modify, via the distortions, the flexural motion and thus the swimming style. The fish-like solid is surrounded by a fluid, salty water, and the interactions between the solid and the fluid generate the propulsive forces that make the fish to move forward. Such simulations require an advanced use of COMSOL; in particular, both moving mesh and remitting are used.

2. Materials and Methods

2.1. Swimming Style

In FSI problems solved with the moving mesh technique, the computational domain Ω is the union of a solid domain Ω_s , assumed as reference configuration for the solid, and a mesh domain Ω_m . Denoted with Ω_{st} the configuration of the solid at time t , the complement $\Omega_f = \Omega/\Omega_{st}$ represents the fluid domain, possibly changing in time; the balance equations for the solid are written with respect to Ω_s (material, or Lagrangian formulation), while those for the fluid are written with respect to Ω_f (spatial, or Eulerian formulation).

State variables are the material field \mathbf{u}_s , describing the displacement of the solid, and the spatial field \mathbf{v}_f , representing the velocity of the fluid. The equations of the problem consist of two equations for the fluid, balance of forces and conservation of mass, and one for solid, the balance of forces:

$$\begin{aligned}\rho_f \dot{\mathbf{v}}_f + \rho_f (\nabla \mathbf{v}_f) \mathbf{v}_f &= \text{div } \Gamma + \mathbf{f} \quad \text{in } \Omega_f \times \mathcal{T}, \\ \dot{\rho}_f + \text{div}(\rho_f \mathbf{v}_f) &= 0 \quad \text{in } \Omega_f \times \mathcal{T}, \\ \rho_s \ddot{\mathbf{u}}_s &= \text{div } \mathbf{S} + \mathbf{f}_v \quad \text{in } \Omega_s \times \mathcal{T},\end{aligned}\tag{1}$$

where ρ_f and ρ_s are the mass density of the fluid and the solid, respectively, and μ is the fluid dynamic viscosity. The fluid is assumed to be linearly viscous; its stress Γ is given by:

$$\Gamma = -p \mathbf{I} + 2\mu_f (\text{sym } \nabla \mathbf{v}_f) - \frac{2}{3}\mu_f (\text{div } \mathbf{v}_f) \mathbf{I}, \quad (2)$$

where p is the fluid pressure. The solid is assumed to be isotropic and linear elastic; its reference stress \mathbf{S} is given by:

$$\mathbf{S} = \mathbf{F}^e \mathbf{S}^e \mathbf{F}^{e*}, \quad \mathbf{S}^e = 2\mu_s \mathbf{E}^e + \lambda \text{tr}(\mathbf{E}^e) \mathbf{I}, \quad (3)$$

with μ_s, λ the Lamé's moduli, $\mathbf{F} = \mathbf{I} + \nabla \mathbf{u}_s$, \mathbf{F}^o the distortions field, $\mathbf{F}^e = \mathbf{F} \mathbf{F}^o{}^{-1}$ the elastic deformation, $\mathbf{E}^e = \mathbf{E} - \mathbf{E}^o$ the elastic strain; \mathbf{E} and \mathbf{E}^o are the non-linear strain measures (Green-Lagrange strains) defined by:

$$\mathbf{E} = \frac{1}{2} (\mathbf{F}^T \mathbf{F} - \mathbf{I}), \quad \mathbf{E}^o = \frac{1}{2} (\mathbf{F}^{oT} \mathbf{F}^o - \mathbf{I}). \quad (4)$$

The muscle actions are modeled by a time-evolving distortion strain $\mathbf{E}^o = \mathbf{E}^o(X, Y, t)$ that produces the sought flexural motion of the fish-like body. Within this approach, a muscle generates motion, possibly without stress; as example, if the imposed distortion is compatible, that is, if a displacement \mathbf{u}_s , such that $\text{sym } \nabla \mathbf{u}_s = \mathbf{F}^o$, can be realized. In our case, the lateral motion of the fish is reacted upon by the force exerted by the surrounding fluid, and any muscles action is accompanied by stress production.

System (1) is supplemented with boundary and initial conditions; $\partial\Omega_{sf}$ is the interface between the fish-like body and the fluid, where FSI boundary conditions are posed:

$$\mathbf{T} \mathbf{n} = -\Gamma \mathbf{n}, \quad \mathbf{v}_f = \dot{\mathbf{u}}_s, \quad \text{on } \partial\Omega_{sf} \times \mathcal{T}, \quad (5)$$

with \mathbf{n} normal to the solid boundary and $\mathbf{T} = \mathbf{S}(\mathbf{F}^*)^{-1}$ the Cauchy stress in the solid; on the boundary of the aquarium $\partial\Omega_a = \partial\Omega_f/\partial\Omega_{sf}$ we assign a no-slip wall condition for the fluid:

$$\mathbf{v}_f = 0 \text{ in } \partial\Omega_a \times \mathcal{T}. \quad (6)$$

Finally, we assign homogeneous initial conditions.

We set our problem in 2D; the reference configuration of the fish-like body is a streamlined region of the XY plane, whose symmetry axis lies in the

X -axis; tail and head are at $(0, 0)$ and $(L, 0)$, respectively. The contour $Y = c(X)$ is given by a mirrored 5th order polynomial function:

$$Y = c(X) = a_1 X^5 + a_2 X^4 + a_3 X^3 + a_4 X^2 + a_5 X. \quad (7)$$

with

$$a_1 = -10.55 \frac{t_s}{L^5}, \quad a_2 = 21.87 \frac{t_s}{L^4}, \quad a_3 = -15.73 \frac{t_s}{L^3},$$

$$a_4 = 3.19 \frac{t_s}{L^2}, \quad a_5 = 1.22 \frac{t_s}{L},$$

where t_s is the maximum fish thickness and L indicates the fish length [4]; the resulting reference configuration is shown in Fig.(1).

Denoting with $Y=h(X,t)$ the transversal displacement of the axis, the relation between distortion \mathbf{E}^o and the curvature of the axis $-\partial^2 h/\partial X^2$ reads as:

$$\mathbf{E}^o_{xx}(X, Y, t) = -Y \frac{\partial^2 h(X, t)}{\partial X^2}. \quad (8)$$

To define a swimming style, we first assign the function $h(X, t)$, and then derive the muscle-driven distortions \mathbf{E}^o_{xx} by using (8).

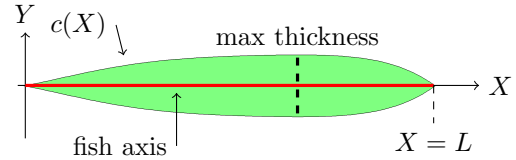


Figure 1: Reference shape of the fish-like body; the contours $\pm c(X)$ are parametrized by eq. 7.

We use as benchmark swimming-style the carangiform style; the transversal displacement of the fish axis is defined as the product of an envelope $e(X)$, a backward-traveling wave, and a time switch:

$$h(X, t) = e(X) \sin(\gamma X + \omega t) (1 - \exp(-t/t_a)). \quad (9)$$

Here, γ is the wave number of body undulations, ω the angular frequency, and $c_o = \omega/\gamma$ the wave velocity; t_a is a characteristic time for activation. The envelope function $e(X)$ describes the maximum lateral displacement of the axis, and is described by:

$$e(X) = \frac{4}{25L} X^2 - \frac{6}{25} X + \frac{1}{10} L, \quad (10)$$

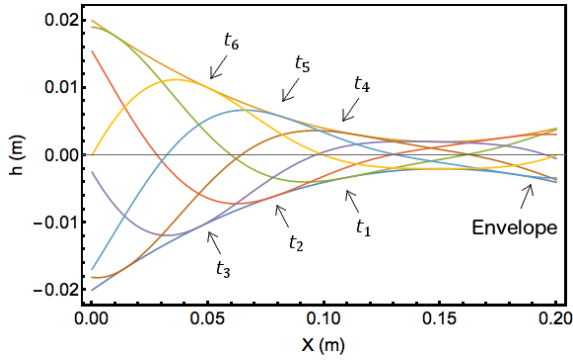


Figure 2: Shape of the fish axis at different times; amplitude of movements is much larger at the tail (left) than at the head (right), and is enclosed in the envelopes. It is present an amplitude wave moving towards the tail, see the position of amplitude maximum at times $t_1 < t_2 < \dots < t_6$; fish motion is rightward.

Fig. 2 shows axis displacement $h(X, t)$ at six different instants $t_i = 2.8 [s] + (i - 1) 0.04 [s]$ with $i = 1, 2, \dots, 6$.

The problem for the solid is solved using the plane strain model; the key model and material parameters are listed in Table 1.

2.2. Post Processing

The two non-dimensional parameters that characterize the steady inline performance of a carangiform swimmer are the Reynolds number (Re) of the flow and the swimming number (Sw) of the undulatory body motion [6], which can be defined as follows:

$$Re = \frac{|\mathbf{v}_{swim}| L \rho_f}{\mu_f}, \quad Sw = \frac{A \omega \rho_f}{2\pi \mu_f}. \quad (11)$$

Fish velocity \mathbf{v}_{swim} is probed at the center of mass, located at $(X, Y) \sim (0.75 L, 0)$; $A \simeq e(0)$ is the maximum lateral excursion of the tail over a cycle. The lift F_L and drag F_D forces are defined, respectively, as:

$$F_L = \int_{\partial\Omega_s} \Gamma \mathbf{n}_f \cdot \mathbf{e}_y ds; \quad F_D = \int_{\partial\Omega_s} \Gamma \mathbf{n}_f \cdot \mathbf{e}_x ds, \quad (12)$$

with $\mathbf{e}_x, \mathbf{e}_y$ the unit vectors of the Cartesian basis.

3. Results

By solving system (1), using data of Table (1), we obtain a carangiform swimming-style as

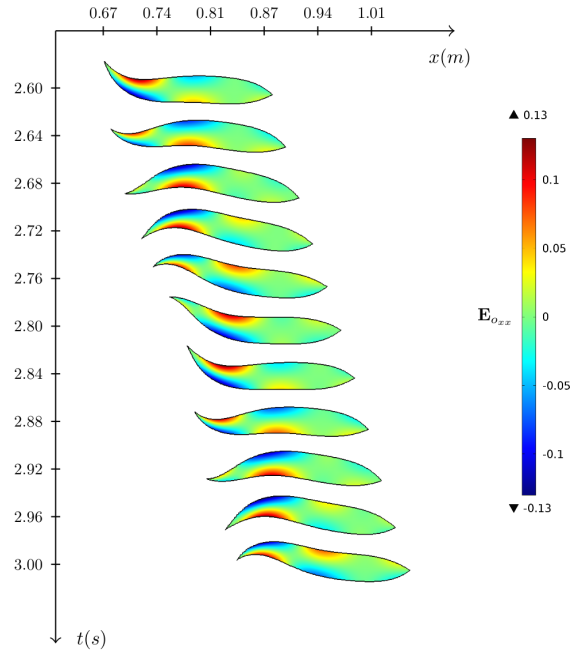


Figure 3: Sequence of shapes assumed by the fish as consequence of muscles activation; color map denotes muscle status: red contracted, blue elongated. This undulatory motion gives the thrust required to swim rightward.

showed in Fig.(3,5). Fish motion can be compared with the expected lateral displacement defined in eq.(9) with optimal axis shape superposition. The rightward fish movement is produced by appropriately tuned distortions field that mimic muscles action: colors denote contraction (red) and elongation (blue). The generation of circular vortex wake can be seen in Fig.(5,6). The fish tail, according to fish such as trout and mackerel [5], appears to function like a propeller, generating a localized thrust wake with an observable momentum jet. The mutual distances between the cores of the vortices do not change. The results are compared with empirical general trends of fish locomotion taken from [1]; in particular, we focus on the relationships between fish length L , amplitude of the undulatory motion of the tail A , speed of the contraction wave traveling along the body c_o , and the realized swimming speed \mathbf{v}_{swim} . Our findings show satisfactory agreement between the model and actual measurements. The relative tail tip amplitude A/L varies between 0.14 and 0.05 L ; usually an average value of 0.10 L is consid-

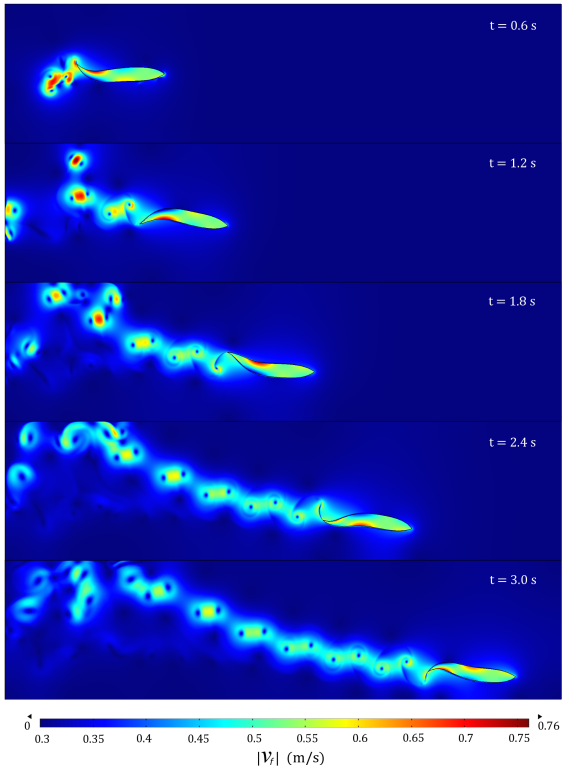


Figure 4: Snapshots of fish swimming with fluid velocity field (color bar denotes fluid speed); muscles stroke (red colored) bends the fish and produces swimming thrust. A long wake lies behind the fish. It is worth saying that swimming is realized in the simulation by tuning the muscles contraction pattern in both space and time.

ered. In our model we get a mean value of $0.138 L$ measured after the first second of fish swimming, when the action of muscles is almost fully generated. The tail displacement can be observed from its trajectory during the simulation that is showed in Fig.(6).

An important relation, outcome of many experimental observations of actual fish swimming, exists between the frequency (f in Hz) and the speed relative to the body length (\mathbf{v}_{swim} in L/s):

$$\frac{|\mathbf{v}_{swim}|}{L} = 0.71 f. \quad (13)$$

This equation provides an accurate first estimate of the speed of any fish, given its length and tail beat frequency; given our model data, eq.(13) yields as expected fish velocity the value $0.56 m/s$. The velocity resulting from our numerical simulation is showed in Fig.(8); the speed achieved

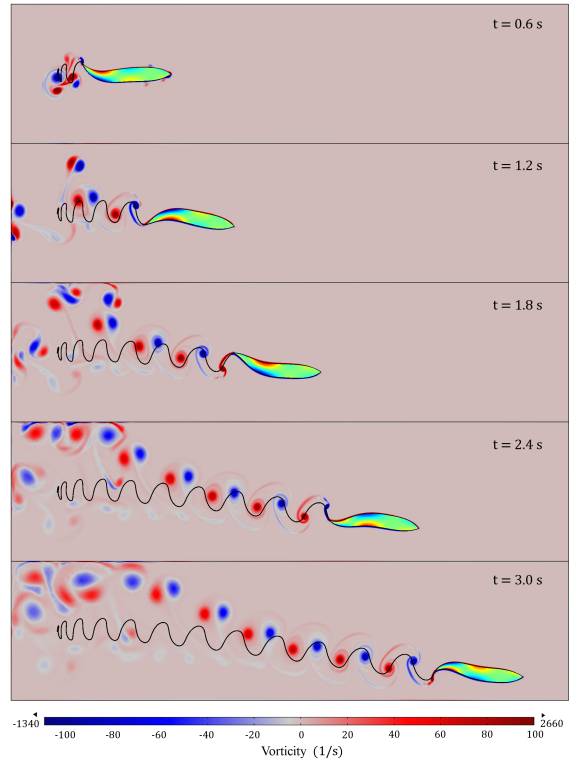


Figure 5: Snapshots of fish swimming with fluid vorticity field; bound vortices, released at the end of every stroke to the left or to the right, are evident. The trajectory of tail tip is black colored; as can be seen, the initial displacements are smaller and closer to each other compared with the final; this is because the fish swimming needs some time to be fully generated.

at steady conditions is about $0.38 m/s$, at time $t = 3.0 s$; we consider this value to be in good accordance with the expected empirical value, provided the fact that our simulations are 2D. The Reynolds number for a fish can be assumed to be proportional to the swimming number [6]:

$$Re \sim S_w. \quad (14)$$

Using eq.(11) we obtain respectively:

$$Re = 7.62 * 10^4 \quad S_w = 1.10 * 10^5 \quad (15)$$

showing another confirm of the model. Finally, we calculate the lift and drag forces on fish body, see Fig.(9). We reach a mean value of $0.21 N$ for lift forces and $0.78 N$ for drag forces. Lift forces have greater oscillation than drag force but with a smaller average such as the velocity components.

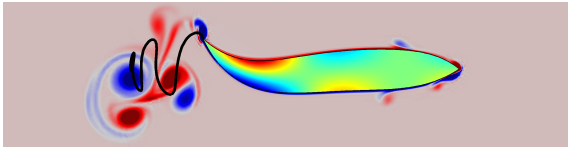


Figure 6: Zoom at the bound vortices released at the initial stage of motion, after two and half stroke cycles.

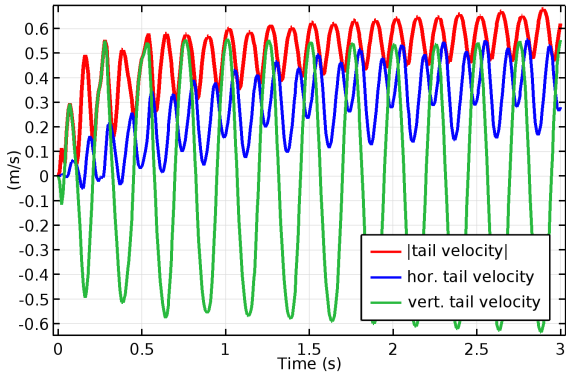


Figure 7: Velocity at the tail tip during the whole duration of simulation; both the vertical component (green) and the horizontal one (blue) exhibit large oscillations, that seems to reach an asymptotic behavior. The contribution to the overall speed (red) comes from both components.

These results are obtained with a maximum absolute distortion $|\mathbf{E}_{xx}^c| = 0.13$, a value compatible with the maximum muscle shortening measured in actual fishes. It is clear that fish shape and its thickness are important influence factors on fish swimming velocity that aren't taken in consideration in this paper.

4. COMSOL Settings

Our approach to simulate fish swimming requires multi-physics, and deeply uses advanced features of COMSOL: non-linear solid mechanics with large distortions to generate fish motion, and fluid mechanics to simulate the Fluid-Solid Interactions (FSI) that makes the fish swim. Moreover, we need both moving mesh to solve the FSI for short time intervals, and re-meshing to track the long swimming path we aim at simulating.

Fish is modeled as a 2D solid undergoing large displacements, and water is modeled as a fluid; the action of fish muscles is modeled in COMSOL by defining a pre-strains field modulated in both

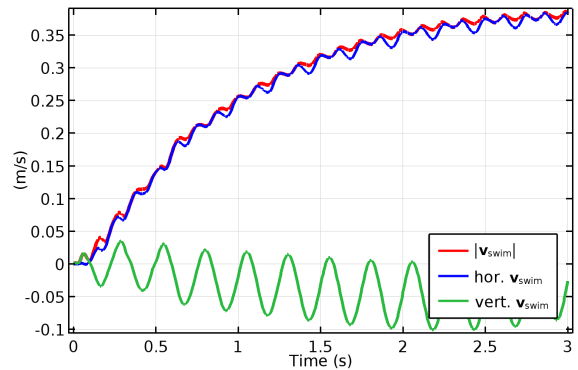


Figure 8: Velocity at the fish center of mass during the whole duration of simulation: the vertical component (green) has large oscillations but small average values, while the horizontal component (blue) has smaller oscillations but a higher average values that seems to reach an asymptotic behavior. The contribution to the overall speed (red) comes essentially from the horizontal part.

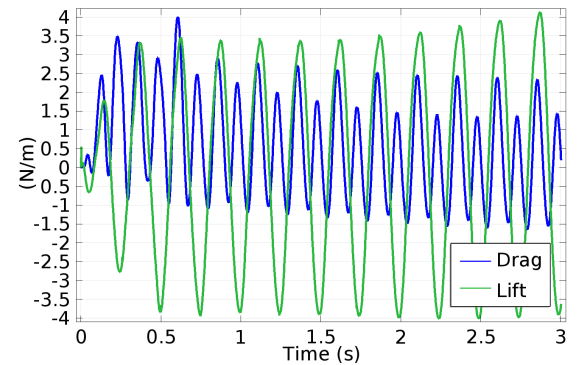


Figure 9: Lift forces have greater oscillation than drag force but with a smaller average such as the velocity components.

time and space. Fish motion yields mesh deformation, and the forward movement of the fish eventually yields to a so large deformation that the automatic re-meshing feature is needed; here, we require `mod1.fsi.I1isoMax < 2` for the maximum element distortion, a classical value after which the element can be considered severely distorted.

BDF (Backward Differentiation Formula) order is also important as it influences the re-meshing of the model; to reduce spurious oscillations produced by remeshing, we set maximum BDF order at 1. It is important to note that the fish is not constrained at all, and that its swimming-like behavior is an outcome of our simulations.

Fluid		
μ_f	0.001 [Pa * s]	fluid dynamic viscosity
ρ_f	1000 [kg * m ⁻³]	fluid density
Solid		
λ	5.4 * 10 ⁵ [Pa]	Lamé 1 st parameter
μ_s	7.7 * 10 ⁵ [Pa]	Lamé 2 nd parameter
ρ_s	1050 [kg * m ⁻³]	solid density
Swimming		
L	0.2 [m]	fish length
t_s	0.03 [m]	fish thickness
γ	31.4 [rad * m ⁻¹]	wave number
ω	25.1 [rad * s ⁻¹]	angular frequency
t_a	0.2 [s]	activation time

Table 1: key model and material parameters.

Time step is 0.001 s, the ratio between mesh size and aquarium size is $\sim 2 \cdot 10^{-3}$, and the whole problem has ~ 0.4 Mdof.

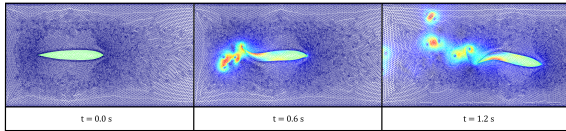


Figure 10: The fish does not have constraints, and swims freely in a large fluid domain; thus, both moving meshes and re-meshing techniques are used to solve the FSI problem. Actually, with moving mesh we deal with the local motion of the fish; with remeshing, we are able to simulate long distance swimming.

5. Conclusions

We proved the feasibility of tackling such a complex problem as fish swimming with COMSOL; our results are encouraging, and provide the basis for further developments.

Acknowledgements The present work is supported by INdAM (the Italian Institute for Ad-

vanced Mathematics) through the Grant Young Researcher 2015.

References

- [1] J. J. Videler. Fish Swimming, Springer-Science+Business Media, B.V., 1993.
- [2] I. Borazjani, F. Sotiropoulos. Numerical investigation of the hydrodynamics of carangiform swimming in the transitional and inertial flow regimes. *The Journal of Experimental Biology* 211, 1541-1558 (2008).
- [3] P. Nardinocchi, L. Teresi. On the Active Response of Soft Living Tissues. *J. Elasticity* 88:2739 (2007).
- [4] J.J. Videler, F. Hess. Fast Continuous Swimming of Two Pelagic Predators, Saithe (*Polachius Virens*) and Mackerel (*Scomber Scombrus*): a Kinematic Analysis. *J. Exp. Biol.* 109:209–228 (1984).
- [5] G.V. Lauder, P.G.A. Madden. Learning from Fish: Kinematics and Experimental Hydrodynamics for Roboticists *International Journal of Automation and Computing* 4 325-335 (2006).
- [6] M. Gazzola, M. Argentina, L. Mahadevan. Scaling macroscopic aquatic locomotion. *Nat Phys* 10:758?761 (2014)