COMSOL CONFERENCE 2019 CAMBRIDGE

# Acoustic Topology Optimization – Implementation and Examples

R. Christensen, PhD
Acoustics Department, GN Hearing A/S
Lautrupbjerg 7, DK-2750 Ballerup, Denmark, rechristensen@gnresound.com; renecmail@gmail.com

**Abstract:** This paper describes the theory behind isentropic acoustic topology optimization and provides several examples to illustrate its use. For example, the technique can convert acoustic modes in a tube; filter sound spatially by frequency content; and help in the design of loudspeaker geometries such as phase plugs and waveguides. Details such as density and projection filters to facilitate binary designs suited for actual production are also discussed, as is the implementation in COMSOL Multiphysics.

**Keywords:** Acoustics, topology optimization, demultiplexer, mode converter, phase plug

## Introduction

Topology optimization is a relatively new discipline in which one or more domains are assigned to be *design domains*; in these domains, one or more material properties are changed via the optimization process, in order to meet certain desired targets.

The techniques has been explored within different physics and applications; arguably most within the framework of structural mechanics around which most of the available literature revolves, see e.g. [1,2]. The physics explored in the present work, however, is *acoustics*, for which much less literature exists [3,4]. In the present work, the theory behind the methodology is described and several examples are given to illustrate its use.

In the topology optimization procedure, one or more design domains is defined in which a single *design variable* can vary throughout an optimization process. Certain *objectives* and *constraints* are defined mathematically for which an optimized design is sought. The design variable controls one or more material parameters. In the case of acoustics, two material parameters are controlled; the density of the fluid and its bulk modulus.

The design variable can vary continuously in a range of 0 to 1, but a binary design is desired, for which the design variable is either 0 or 1. This is typically achieved by having certain *material interpolation* functions describing the relationship between the design variable and the material parameters, often accompanied by incorporating certain *projection filters* further forcing *binary designs*:
A binary design is typically visualized with white and black sections in the design domain; for acoustics, white indicates a design variable of 0, which in turn describes a fluid with air properties. Black sections indicate a design variable of 1 and represent a heavy and stiff fluid; its boundaries are typically assumed replaceable with a hard wall boundary condition. However, for certain cases, a design with this boundary condition will not give the same result as that indicated by the initial optimization routine, so one should always test the optimized design with the hard wall approach, as this is how the end design will ultimately be constructed. This test has been carried out for the design examples shown in this paper to ensure that the designs work as intended.

## Theory

In the design domain(s) a design variable $\xi$ is defined via [1]

$$0 < \xi(\mathbf{x}) \leq 1 \quad \forall\, \mathbf{x} \in \Omega_\mathrm{d}$$

where $\mathbf{x}$ denotes the coordinate space and $\Omega_\mathrm{d}$ is the design domain.

In order to avoid numerical instabilities, such as mesh dependency and checkerboarding [1], a *density filter* can be applied. This can be implemented via a Helmholtz-type partial differential equation [5] as

$$-\mathrm{r}^2\nabla^2\tilde{\xi} + \tilde{\xi} = \xi, \qquad \frac{\partial\tilde{\xi}}{\partial \mathrm{n}} = 0$$

with a resulting filtered design variable $\tilde{\xi}$. The filter radius r is adjusted to fit with e.g. two mesh element side lengths, depending on the problem at hand.

As a means of forcing binary (black and white exclusively) designs without grey areas, it can be advantageous to apply a so-called *projection filter*, for which the design variable, or the density filter variable if one such filter is applied, is projected onto a new design variable axis in order to ensure that grey designs go towards either black or white. This is mathematically defined as [6]

$$\bar{\xi} = \bar{\xi}\left(\tilde{\xi}(\xi)\right) = \frac{\tanh(\beta\eta) + \tanh\left(\beta(\tilde{\xi} - \eta)\right)}{\tanh(\beta\eta) + \tanh(\beta(1 - \eta))}$$

for the projection filtered (and here assuming also density filtered) design variable $\bar{\xi}$. The parameters $\beta$ and $\eta$ determine the projection filter characteristics.

A *continuation scheme* was applied for some of the examples shown later for which the $\beta$-value was ramped up in steps during the optimization process [6]. A continuation scheme for the density filter radius has also been suggested [7] but was not found necessary for the examples shown in this work. Also, *sensitivity filtering* [8] was not considered here.

It is assumed that the acoustic process is isentropic, i.e. no losses are included the optimization, at least in the design domain. In this case, the complex harmonic pressure p is found via the Helmholtz equation based on the projected design variable as

$$\nabla \cdot \left(\frac{1}{\rho(\bar{\xi})} \nabla \mathrm{p}\right) + \frac{\omega^2}{\mathrm{K}(\bar{\xi})} \mathrm{p} = 0.$$

with the density ρ, the bulk modules K, and an angular frequency of ω. The material values for the acoustic medium in the design domain are modified according to the value of the design variable using an interpolation scheme such as the Solid Isotropic Material with Penalization (SIMP) scheme. Here, intermediate values of the design variable are *penalized* so that they seek towards being closer to 0 or closer to 1 as the optimization is carried out, in order to have less grey in the design domain. A SIMP scheme is mathematically described as [1]

$$\rho(\bar{\xi}) = \rho_0 + \bar{\xi}^{\mathrm{P}}(\rho_{\mathrm{s}} - \rho_0)$$

where $\rho(\bar{\xi})$ is the density as a function of the design variable, $\rho_0$ is the density for air, $\rho_s$ is a density value for a very dense fluid, and P is a penalization constant; the higher the value, the higher the penalization.

For a (possible density and projection filtered) design variable value of 0, the density $\rho(\bar{\xi})$ equals the standard $\rho_0 = 1.2\ \mathrm{kg/m^3}$ for air, whereas a design variable of 1 relates to a very dense fluid; e.g. with $\rho_\mathrm{s}$ set to $1{,}000\ \mathrm{kg/m^3}$.

Intermediate values are penalized since they have very similar resulting material values as a design variable value of 0 (=air) yet take up a substantial amount of the available integrated design variable, and so in order to comply with the volume ratio constraint the solver seeks away from such intermediate values.
An equivalent scheme with appropriate values is defined for the bulk modulus K of the fluid in the design domain, ensuring a very stiff medium for a design variable of value 1 (=solid).
For certain problems, alternative interpolation schemes are advantageous, such as e.g. the Rational Approximation of Material Properties (RAMP) scheme [9], which was used for several of the examples here.

A critical part of the optimization process is defining proper objective functions for which a solution is sought. This will often be an iterative process, as the resulting design often reveals inadequacies of the objection function/functions, and so it/they must be refined or reformulated. Oftentimes an extra objective function is needed with a weight to scale it to the original objective function, to avoid trivial solutions, such as closing off an input entirely in order to minimize the output. A mathematical formulation of the acoustic optimization problem can typically be written as a minimization problem as

$$\begin{gathered}\min_{\xi}: \Phi\left(\mathrm{p}\left(\bar{\xi}(\xi)\right), \xi\right) \\ \text{subject to: } \mathrm{c_i}\left(\mathrm{p}\left(\bar{\xi}(\xi)\right), \bar{\xi}\right) \leq 0, \qquad i \in \{1,2, \dots\}\end{gathered}$$

where Φ is an objective function, and p is the complex acoustic pressure in the design domain. Several constraints $c_i$ limit the design space. A typical constraint is a volume fraction constraint, which limits how much of the design domain can be assigned a design variable of 1, indicating a structural domain. Such a constraint is sometimes included, even if not physically needed, for improved numerical convergence, but oftentimes one is of course limited in how much of the domain can be taken up by a rigid structure. The volume fraction constraint is defined via

$$\frac{\int_{\Omega_d} \bar{\xi}\, d\Omega_d}{\Omega_{max}} - 1 \leq 0$$

where a predefined maximum allowable part of the design domain, denoted $\Omega_{max}$, can be assigned a design variable value of 1.

Sensitivities are found via an adjoint method [10], and the Method of Moving Asymptotes (MMA) [11] is used for solving the optimization problem.

## Implementation in COMSOL Multiphysics[1]

Several parameter values, e.g. for the interpolation extremes, are input under **Parameters**.

The material interpolations for density and bulk modulus, respectively, are input under **Variables** as functions of the design variables. These function variables are used as input in a separate **Pressure Acoustics** node for the design domains only.

A **Topology Optimization** node is added under **Definitions**, with a **Projection** applied using a Projection Slope value initially defined under Parameters, but its value is often controlled during the **Study**.

An **Optimization** node is added from the **Mathematics** selection, and typically one or more **Global Objectives** are added. Also, the allowed volume fraction is controlled via a **Global Inequality Constraint**.

The **MMA solver method** is chosen, with the **Adjoint gradient method** activated.

## Examples

The following examples have been chosen to show the applicability of the acoustic topology optimization scheme described in the above, with a special focus on cases for which non-intuitive designs emerge.

### Criss-cross Splitter, 2D

The *Criss-cross Splitter* can have sound travel from one input crossing a domain to an output, whereas for a second input sound can travel to another input crossing the design domain along the other diagonal. The geometry is shown in Figure 1; the side lengths of the design domain are all 1 meter.

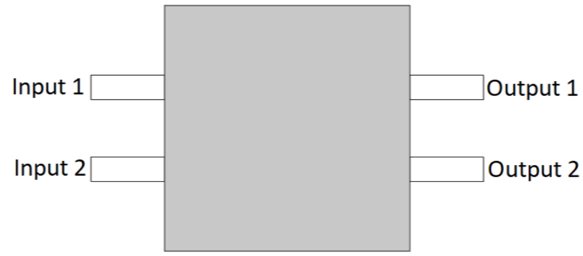


Figure 1: The Criss-cross Splitter is intended to make one frequency input cross the design domain shown in grey from Input 1 to Output 2, while another frequency input crosses from Input 2 to Output 1.

Plane wave boundary conditions are applied at all inputs and outputs, with a plane wave pressure amplitude with differing frequencies at each input.

The objective function can be written as a max-min problem as

$$\max_{\bar{\xi}} : \min_{f_{i,j}} \frac{I_{out,i}(\bar{\xi}, f_{i,j})}{I_{in,j}(\bar{\xi}, f_{i,j})}$$

to optimize the intensity transfer $I_{out}/I_{in}$ for each of the two transfer paths at their respective assigned frequency, i.e. $f_{i,j} = f_{1,2}$ and $f_{i,j} = f_{2,1}$, respectively.

The resulting topology optimized design is shown in Figure 2 for $f_{2,1} = 900$ Hz related to the Input2-to-Output1 transfer, and $f_{1,2} = 1{,}260$ Hz related to the Input1-to-Output2 transfer.

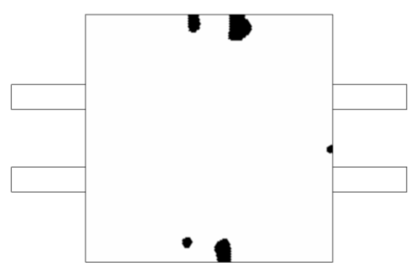

Figure 2: The topology optimized design for $f_{2,1}$=900 Hz (Input 2-to-Output1) and $f_{1,2}$=1,260 Hz (Input1-to-Output2).

The resulting acoustic pressure is shown in Figure 3 with arrows indicating the acoustic intensity. The 'criss-cross' nature of the setup is clearly seen. The superposition principle dictates that this functionality be present when both inputs are active simultaneously.

[1] This section assumes implementation in v5.4.

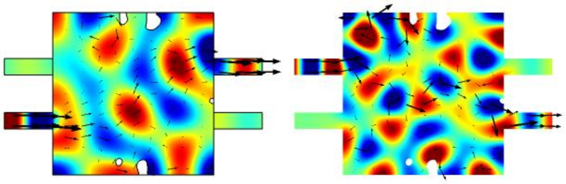

Figure 3: The acoustic pressure topology optimized design for a frequencies $f_{2,1}$=900 Hz (Input2-to-Output1) shown on the left and $f_{1,2}$=1,260 Hz (Input1-to-Output2) on the right. Arrows indicate acoustic intensity.

## Acoustic Demultiplexer, 2D

The *Acoustic Demultiplexer* can split a single input signal to three outputs depending on the input frequency, as a sort of 'acoustic cross-over filter' in contrast to a normal electrical cross-over filter found in most loudspeakers. It is assumed that only three discrete frequencies can occur at the input, but it is also possible to formulate band-pass regions for each output. The geometry is shown in Figure 4 with the design domain indicated with grey color. Plane wave boundary conditions are applied at the input and all outputs; for the input a plane wave amplitude is also applied.

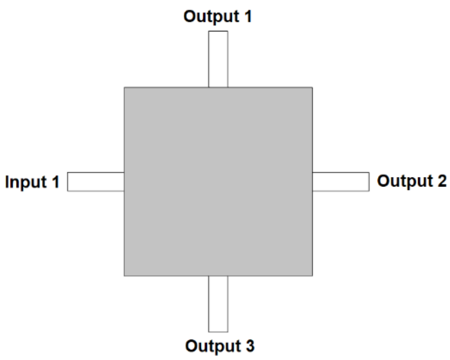


Figure 4: The Acoustic Demultiplexer with a design domain shown in grey is intended to split a single input into three outputs depending on the input frequency.

The objective function can be written as a max-min problem as

$$\max_{\bar{\xi}} : \min_{f_i} I_{out,i}(\bar{\xi}, f_i)$$

essentially seeking to even out differences between the three output intensities $I_{out,i}$ at their respective assigned frequencies.
For the frequencies $f_1 = 800$ Hz, $f_2 = 960$ Hz, and $f_3 = 1{,}120$ Hz, and a design domain of $1\ m^2$, the resulting topology optimized design is shown in Figure 5.

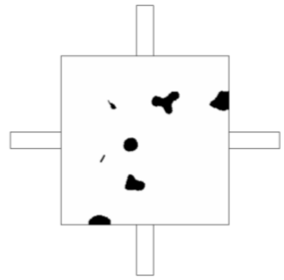

Figure 5: The optimized design for the given frequencies and a 1 $m^2$ design domain.

The black domains are replaced with hard wall boundary conditions on their edges, and the resulting pressure field is shown in Figure 6, where the arrows indicate the acoustic intensity. It is clearly seen that the Acoustic Demultiplexer with one fixed design can split the input frequency content into individual outputs.

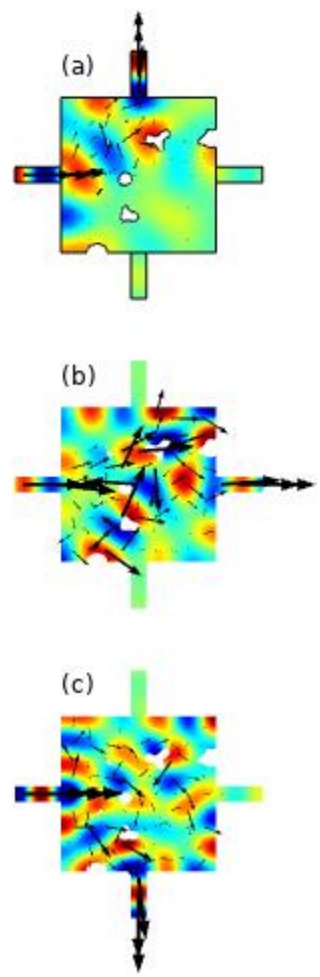


Figure 6: The resulting pressure distribution in the Acoustic Demultiplexer for three different frequencies at the input: (a) 800 Hz, (b) 960 Hz, and (c) 1,120 Hz. The side lengths of the design domain are each 1 m. Arrows indicate acoustic intensity.

**Acoustic Mode Converter, 3D**

Acoustic modes in ducts are an interesting topic which is important for many applications. A duct has an infinite number of geometrically inherent modes. The total sound field in a duct can be described by its modes, but the input excitation will dictate to which degree modes are present, and the frequency will determine if a mode is propagating or evanescent. If the cross-section is constant, the modes will not change their characteristics spontaneously. However, with acoustic topology optimization part of the tube geometry can be modified, so that certain modes are converted into other modes. This can in certain cases be a remedy for undoing other effects coming from geometry changes such as kinks in/bends on the tube. In other cases, one might want to turn an evanescent wave into a propagating one, or vice-versa.

The functionality is exemplified via a three-dimensional case; the *Acoustic Mode converter*. It starts out as a model found in COMSOL's application library[2], slightly modified, and topology optimization added. A rectangular tube is exited at one end with a plane wave, and due to a kink and a bend in the tube the output has a multimodal behavior above a certain frequency, as illustrated in Figure 7.

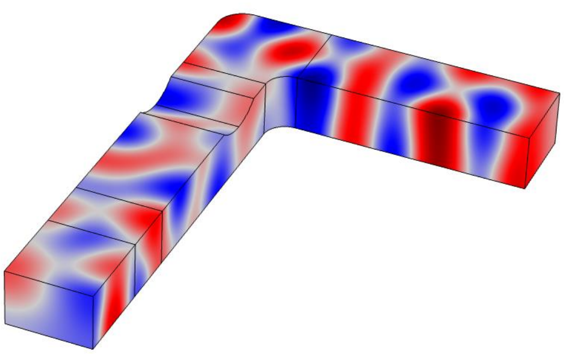

Figure 7: The acoustic pressure for the initial geometry, and the resulting sound pressure for a plane wave input at 1,180 Hz.

The plane wave would have been the only mode at all frequencies for the excitation in question were it not for the bend and the kink, but with acoustic topology optimization it is possible to negate the effect of the kink and the bend and restore the plane wave propagation at the output and onward.

The design domain was chosen to be the corner geometry; the bend.

[2] Search for "Duct with Right Angled Bend".

The objective function is expressed for a single frequency input as

$$\min_{\bar{\xi}}: \sqrt{\mathrm{aveop}_{\mathbf{x}_{\mathrm{out}}}\left(\left(\mathrm{p}(\bar{\xi})\mathrm{p}^{*} - \mathrm{aveop}_{\mathbf{x}_{\mathrm{out}}}\left(\mathrm{p}(\bar{\xi})\mathrm{p}^{*}\right)\right)^{2}\right)}$$

minimizing the acoustic pressure variance over the output surface with coordinates $\mathbf{x}_{\mathrm{out}}$, pressure and complex conjugated pressure p and $\mathrm{p}^*$, respectively, utilizing an average operator over the output surface denoted $\mathrm{aveop}_{\mathbf{x}_{\mathrm{out}}}$.

The acoustic pressure for the topology optimized is shown in Figure 8.

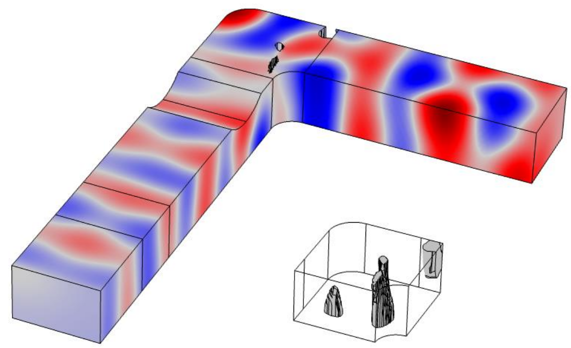

Figure 8: The acoustic pressure for the topology optimized geometry, restoring a plane wave output despite the bend and the kink in the rectangular tube for a plane wave input at 1,180 Hz. The optimized geometry is also shown.

The mode conversion has been generalized, such that one non-planar mode can be converted to another non-planar mode, be they evanescent or propagating. This is illustrated in Figure 9 for a (0,1)-to-(0,2) radial mode conversion, both modes propagating, in a circular cylindrical tube.

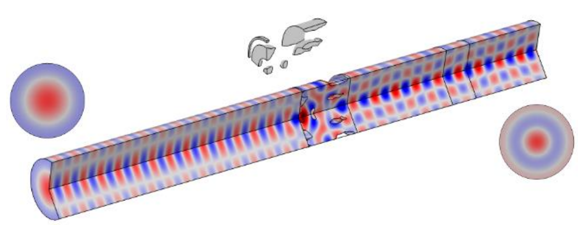

Figure 9: A (0.1) mode on the left input of the circular cylindrical tube is converted to a (0.2) mode after the insertion of a topology optimized center piece. A quarter revolution of the optimized solid geometry is also shown.

## Acoustic Cloaking, 2D

Topology optimization can also be used for *acoustic cloaking* as illustrated below. The example starts out as an application library file[3], on top of which topology optimization is added. A plane wave input is scattered due to a semicircular obstruction, see Figure 10. With topology optimization applied to two inner domains and an objective in the two outer domains of

$$\min_{\bar{\xi}}: \int_{\Omega} SPL_{Scattered}\, \mathrm{d}\Omega$$

an optimized geometry can be found as seen in Figure 11, where the plane wave characteristics have somewhat been restored, thus cloaking the obstruction.

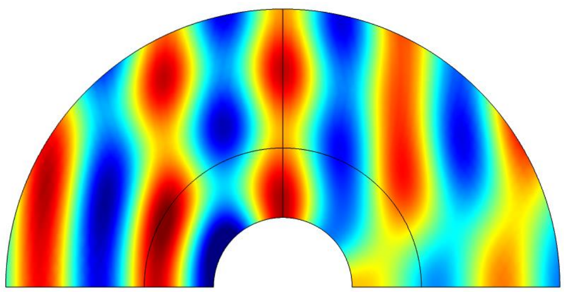

Figure 10: A semicircular cutout causes a scattered sound pressure field for a incoming plane wave going left to right.

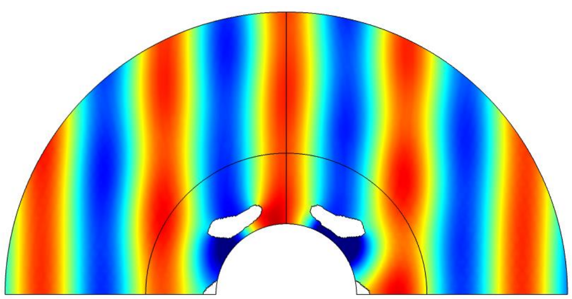

Figure 11: The topology optimized geometry and the resulting sound field for a plane wave going left to right.

## Tweeter Phase Plug, 2Daxi

A final example of acoustic topology optimization is a *phase plug* for a tweeter. The tweeter was built to be as realistic as possible, when it comes to geometry, material parameters and electromagnetic properties. An initially empty design domain was defined in front of the speaker, and the initial on-axis frequency response was found to have a valley across the frequency range of interest. A flatter response was sought via topology optimization. The objective is formulated as a least-squares problem as

$$\min_{\bar{\xi}}: \frac{1}{\mathrm{n}} \sum_{\mathrm{i}=1}^{\mathrm{n}} \sqrt{\left(\mathrm{SPL}(\bar{\xi}, \mathrm{f_i}) - \overline{\mathrm{SPL}}(\mathrm{f_i})\right)^2}$$

for n frequencies of interest and a desired sound pressure level response $\overline{\mathrm{SPL}}$.

The topology optimized geometry is shown in Figure 12 along with the resulting sound pressure. The optimization was carried out as a 2D axisymmetric case, but a quarter revolution view is shown in the figure. The rectangular design domain is also shown.

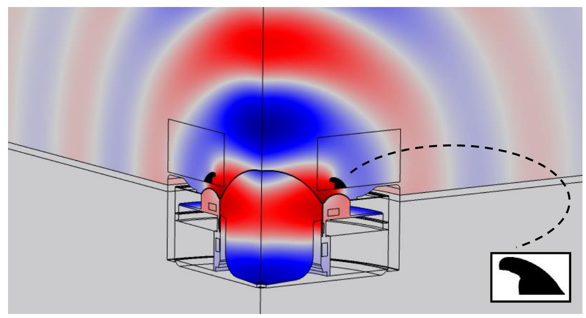

Figure 12: The resulting topology optimized phase plug is shown in black along with a zoom (right-bottom corner) of the phase plug cross-section, and with the resulting sound pressure inside and outside the tweeter at 16 kHz.

The initial and the optimized on-axis sound pressure levels are shown in Figure 13, where it is seen that the sound pressure level has been flattened across the frequency range of interest after the optimization.

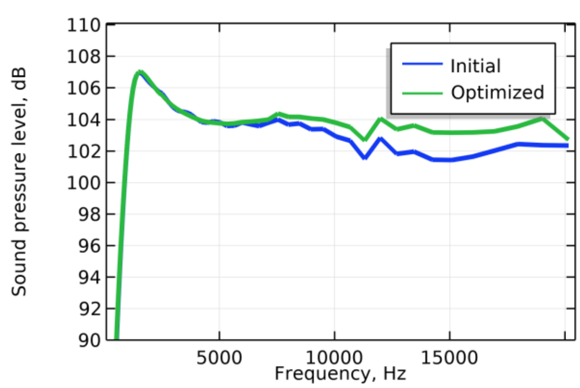


Figure 13: On-axis pressure from the tweeter in linear frequency scale, before (blue) and after the optimization (green) procedure having creating the phase plug geometry.

---

[3] Search for "Acoustic Cloaking".

**Discussion**

For each optimization setup there always is an initial design, i.e. a chosen design variable value in the design, for example 0.5 throughout the domain. The final design is typically sensitive to this choice, but for the shown examples the optimization scheme was quite robust, so that the initial design would not significantly change the optimized geometry.

Filtering was applied on a case-by-case basis. The user must experiment with this for his or her application in order to get confident with the effect of filtering.

The volume constraint was applied for all cases, and different values were tried, before settling on a value suited for the example in question. As mentioned in the Theory chapter, even if there are no specific volume constraints for the physical setup, it is often advantageous to include one such constraint for the sake of the numerical solution process.

The robustness of a topology optimized design can be an issue; if the acoustic performance is highly sensitive to manufacturing tolerances, the design is not robust, and so additional measures must be introduced to the optimization process. This topic is beyond the scope of this work, but robustness was considered for the test cases.

## Conclusion

The current paper illustrates how COMSOL Multiphysics can be used for doing topology optimization for acoustics applications. A design domain is defined for which a design variable is continuously modified in the optimization process to describe the evolving design, but a binary design is sought in the optimization process. As the design variable is related to the density and the bulk modulus of the fluid medium, the end design is divided into sections with air properties, and sections that are given hard wall conditions on their boundaries.

Several examples have been provided to show what can be achieved with acoustic topology optimization. It is seen that topology optimization can provide non-intuitive solutions, which would not have been found via a user-driven trial-and-error process. Hence, the method can serve as inspiration to new design paths, and in certain cases it can directly lead to an end product design, if manufacturing constraints have been properly included. There is great potential for this method as it evolves, and it is likely to become a powerful tool in any future engineer's toolbox.

## References


1. M.P. Bendsøe, O. Sigmund, *Topology Optimization*, Springer, New York (2004)
2. M.P. Bendsøe, N. Kikuchi, Generating optimal topologies in structural design using a homogenization method, *Computer Methods in Applied Mechanics and Engineering*, **71**(2), 197-224 (1988)
3. M.B. Düring, J.S. Jensen and O. Sigmund, Acoustic design by topology optimization, *Journal of Sound and Vibration*, **317**, 557-575 (2008)
4. R. Christiansen, Topology Optimization for Wave Propagation Problems with Experimental Validation, 2016, PhD thesis, DTU Mechanical Engineering.
5. B.S. Lazarov, O. Sigmund, Filters in topology optimization based on Helmholtz-type differential equations, *Int. J. Numer. Methods Eng.*, **86**(6), 765-781, (2011)
6. F. Wang, B.S. Lazarov, O. Sigmund, On projection methods, convergence and robust formulations in topology optimization. *Structural Multidisciplinary Optimization*, 43:767-784, 2011
7. N. Aage, E. Villads, Topology optimization of microwave waveguide filters, *Int. J. Numer. Meth. Engng.* 2017; **00**:1-29
8. O. Sigmund, On the design of compliant mechanisms using topology optimization. Mechanics of Structures and Machines 25(4):493–524, 1997
9. M. Stolpe, K. Svanberg. An alternative interpolation scheme for minimum compliance topology optimization. *Structural and Multidisciplinary Optimization*, 22(2):116–124, 2001.
10. J.S. Jensen, Adjoint sensitivity analysis for linear dynamic systems with time-harmonic excitation, Centre for Acoustic-Mechanical Micro Systems, Technical University of Denmark, 2017
11. K. Svanberg, The Method of Moving Asymptotes – A New Method for Structural Optimization, *Int. J. Numer. Methods Eng.*, **24**, 359-373, (1988)