Design and Optimization of a High Performance Ultrasound Imaging Probe Through FEM and KLM Models

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Abstract: The design and optimization of high performance ultrasound imaging probes often needs a reliable model to provide performance estimates related to project upgrades. Finite Element Model (FEM) is the most powerful and comprehensive tool available for this application, but needs a very fine preliminary tuning procedure in order to obtain consistent results. This was described in our previous work [1], where we considered a simple disk transducer layout and outlined an appropriate model optimization procedure.

The present work takes a step forward, approaching the development of a full FEM model for linear array high performance ultrasound transducer, consisting of 144 piezo–elements, with array pitch as low as 245 µm. The large number of parameters that need to be determined to model the array transducer led us to consider a preliminary design procedure, before getting into the FEM development.

The simplified model of the transducer consists in a mono dimensional electro–acoustical KLM model, along with an equivalent transmission line for the "matching layers", that proved to be a very useful design tool. Once the starting values for the principal parameters of the transducer were calculated, the FEM was developed and optimized. Eventually the optimized transducer was manufactured, so that agreement between transducer measured performances and simulation results were checked and both KLM and FEM models were validated.

The results presented here show that the optimized model predicts measurement results in term of electrical impedance magnitude and emitted sound pressure level frequency response.

Moreover, the FEM allows to simulate the important properties of directivity and beam steering capability of the transducer, and their dependency on materials" parameters can be analyzed.

1. Introduction

Imaging probes for diagnostic ultrasonography are devices that generate a pressure field into the human body, according to an electrical signal [2]. The differences in acoustic properties of different types of tissue allow the scanner to generate an image of a part of the body, based on the echo signals. The quality of the resulting image is strictly related to the technology level of the materials involved in the transducer manufacturing and the understanding of their interactions. This is why a complete Finite Elements Model (FEM) for such a device can greatly help in the study and optimization of its electro–acoustical performances [3].

In the present work, a wide band 5 MHz linear array probe (operating from 2 MHz up to 11 MHz measured at −20 dB bandwidth) consisting of an array of 144 piezoelements with 245 µm pitch is designed and manufactured. The transducer design consists in a specialized piezocomposite material, a hard rubber backing substrate, four acoustic matching layers and silicon rubber lens.

Our work was focused on four main points:

1) The use of the KLM model [4] to obtain a *rough fulfillment* of the single array element required performances (i.e. bandwidth). This was achieved by a preliminary determination of the starting values for the principal parameters involved in the model.

2) The development of the FEM and the corresponding optimization procedure needed to obtain a *complete fulfillment* of the required performances. This was achieved by following the *"step approach"*, described in our previous work [1].

3) The manufacturing of the optimized transducer and the comparison between the

single array element measured performances, in term of electrical impedance magnitude and emitted pressure frequency response, were compared with the simulation results, validating both KLM and FEM models.
4) The simulation of important probe properties such as emitted beam directivity and "*beam steering*" capabilities.

2. KLM, FEM: Governing Equations

We briefly describe here the essential model characteristics and the corresponding governing equations used for KLM and FEM model.

2.1. KLM model

The equivalent network of a thickness–mode piezoelectric transducer can be represented by the KLM model as shown in Figure 1, where the "electric" part is composed of clamped capacitance C_0 and a second reactive term jX_1. The "mechanical" part of the KLM circuit is equivalent to a lossy acoustical transmission line, and ϕ is the transformer ratio of electric voltage to mechanical force.
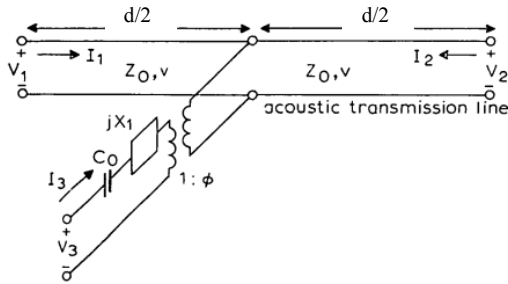

Figure 1: equivalent KLM network of a thickness–mode piezoelectric transducer, with thickness „d".

The circuit parameters of the piezoelectric material are defined as follows:

$$\begin{cases} C_0 = \dfrac{\varepsilon_{33}^s A}{d} \\ Z_0 = \rho v A \\ X_1 = \dfrac{h_{33}}{\omega^2 Z_0} \sin\left(\dfrac{\omega d}{v}\right) \\ \phi = \dfrac{\omega Z_0}{2 h_{33}} \dfrac{1}{\sin\left(\dfrac{\omega d}{2v}\right)} \end{cases} \quad (1)$$

where ε_{33}^s is the permittivity coefficient under no applied voltage (zero strain), h_{33} is the piezoelectric pressure constant, ρ is the density, v is the speed of longitudinal sound waves, A and d are respectively the surface area and the thickness, Z_0 is the acoustic impedance and ω is the angular frequency.

Once the piezoceramic transducer model has been implemented, acoustical matching layers have to be considered. The acoustical transmission line is equivalent to an electrical transmission line [5], with the following relation:

Electrical system		Acoustical system	
voltage	V	force	F
current	I	displace-ment	D
characte-ristic	Z_0	characte-ristic	Z_a
impedance	$= \sqrt{L/C}$	impedance	$= \rho v_a A_a$
phase	v_p	phase	v_a
velocity	$= \sqrt{1/LC}$	velocity	$= \sqrt{C_e/\rho}$
inductance	L	mass	ρA_a
	$= Z_0/v_p$		$= Z_a/v_a$
capacitance	C	compliance	$1/(\rho v_a A_a v_a)$
	$= 1/(v_p Z_0)$		$= 1/[\rho A_a(C_e/\rho)]$
			$= 1/(C_e A_a)$
resistance	R	loss	R'
	$= \omega L \delta$	factor	$= \omega \rho A_a \delta_m$

where δ and δ_m are loss factors of electrical and acoustical waves while C_e is the elastic constant.

The four acoustic matching layers (with acoustic impedance Z_1, Z_2, Z_3, Z_4) were been implemented as in Figure 2:
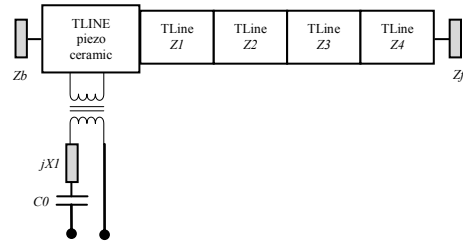

Figure 2: complete equivalent KLM model of the ultrasound transducer, with backing and 4 matching layers.

Where Z_f is the acoustical impedance of the biological medium where the transducer is applied (~1.5 MRayls) and Z_b is the acoustical impedance of the backing material (7 MRayls).

Based on the previous considerations, it was possible to use the electrical transmission line matching theory [6], [7] (i.e. maximally flat or Chebyschev responses) in order to determine the values of matching layers" acoustic impedance and thickness that give the desired performances for the probe.

2.2. Piezoelectricity equations in COMSOL

The constitutive equations for a piezoelectric material are [8], in *stress–charge* form :

$$\begin{cases} \mathbf{T} = \left[\mathbf{c^E}\right]\mathbf{S} - \left[\mathbf{e}^t\right]\mathbf{E} \\ \mathbf{D} = \left[\mathbf{e}\right]\mathbf{S} + \left[\mathbf{\varepsilon^S}\right]\mathbf{E} \end{cases} \quad (2)$$

where $\mathbf{T}$ is the stress vector, $\mathbf{c}$ is the elasticity matrix, $\mathbf{S}$ is the strain vector, $\mathbf{e}$ is the piezoelectric matrix, $\mathbf{E}$ is the electric field vector, $\mathbf{D}$ is the electric displacement vector, $\mathbf{\varepsilon}$ is the dielectric permittivity matrix. The superscripts indicates a zero or constant corresponding field. Equations (2) takes into account piezoelectricity, mechanical and electrical anisotropy of the material.

Once these matrices have been specified, COMSOL recognizes which equations domains are to be used inside the FEM elements.

2.3. Acoustics equations in COMSOL

Pressure waves emitted from the piezoelectric transducer in a biological medium are solution to the wave equation (time domain):

$$\nabla^2 p(r,t) - \frac{1}{c^2}\frac{\partial^2 p(r,t)}{\partial t^2} = 0 \quad (3)$$

where $p(r,t)$ is the pressure and c is the speed of sound in the medium.

It is possible to identify two significant regions where wave propagation characteristics are very different: near field and far field region [2]. As regard our application, the region of interest is generally the far field, where waves are locally planar and the pressure amplitude drops at a rate inversely proportional to the distance from the source.

For homogeneous media, the solution of (2) can be written as a boundary integral (*Helmholtz–Kirchhoff*) anywhere outside a closed surface S containing all sources, in terms of quantities evaluated on the surface. If one takes the limit for distances of observation much greater than the surface extension, then the integral becomes ([8], 2D case):

$$p_{far}(R) = \frac{1-j}{4\sqrt{\pi k}}\int_S e^{ik\frac{r\cdot R}{|R|}}\left(\nabla p(r) - jkp(r)\frac{R}{|R|}\right)\cdot n\, dS \quad (4)$$

where k is the wave number, $\mathbf{R}$ is the vector position of observation point, $\mathbf{r}$ is the vector position of source point and $\mathbf{n}$ is the normal vector pointing into the domain that S encloses.

Starting from (3), if the transducer top face corresponds to the integration surface S, placed in $y=0$, we can express the far field sound pressure p_{far} as:

$$p_{far}(X,Y) = \frac{1-j}{4\sqrt{\pi k}}\int_{y=0} 2\cos\left(\frac{kxX}{\sqrt{X^2+Y^2}}\right)\cdot H(X,Y)\, dx \quad (5)$$

where X, Y is the position of observation point, x is the position on surface S and:

$$H(X,Y) = \frac{dp}{dy}\bigg|_{(x,0)} - jk\, p\big|_{(x,0)}\frac{Y}{\sqrt{X^2+Y^2}} \quad (6)$$

This integral has been used in COMSOL as integration variable, in order to get an analytic expression for the far field pressure emitted by the transducer.

Besides, the far field sound pressure level (*SPL*) has been used, defined as ratio of p_{far} to the reference value (p_{ref} = 20 µPa in air) and expressed in *dB*.

2.4. Electric equations in COMSOL

The electrical impedance Z of a piezoelectric plate can be calculated (by using *Ohm* law), from the potential difference V and the flowing current I across the plate faces. The latter is calculated as surface integral of the current density component along y axis and can been used in COMSOL as integration variable, in order to use the optimization module with objective function given by the difference of measured and simulated electrical impedance.

3. Linear array probe design and manufacturing

In the following paragraphs we briefly describe the manufacturing procedure of the array transducer and its characterization, making use of a dedicated testing equipment.

3.1. Transducer assembly

Ultrasound imaging probes typically consist of an array of piezoelements (with sub–mm pitch), surrounded by special material structures with carefully optimized acoustical properties. The piezoelectric material chosen for the linear

array probe is a 2–2 piezocomposite (PZT–epoxy) having a lower acoustic impedance value with respect to standard PZT ceramic [9] (~ 22 MRayls vs. ~ 30 MRayls).

Following the standard manufacturing procedure, the transducer structure can be described as follows: as first step, the piezoelectric material must be soldered or glued to a conductive fishbone that will provide the electrical connections to the PCBs that send and receive signals from the scanner, through the probe cable. Then, the piezoelectric material is bound on a backing substrate material which acts not only as a support, but also as an efficient damper for the back–traveling pressure wave: we've used an high density rubber, loaded with metallic powder, with acoustic impedance of 7 MRayls. On the front, two or more "matching layers" are glued on top of the piezoelectric material, to allow maximum transfer of power between high impedance and low impedance acoustic medium (i.e. human tissue, ~1.5 MRayls). The matching layers were manufactured from special epoxy resin, charged with different quantities of tungsten fine powder, in order to get the desired acoustic impedance and speed of sound values.

All these materials are modeled in COMSOL as isotropic elastic materials, except for the piezomaterial (see §2.2). The assembly is finally diced with special dicing machines, to get the array structure, and the dicing kerfs are filled with polyurethane. The total number of array elements can be from 96 to 192 or even more, depending mainly on the imaging probe application. In the case of the probe presented in this work, the number of elements is 144. Figure 3 and 4 are pictures of the transducer array, without and with silicon lens.

Figure 3: Transducer assembly.

Figure 4: Complete probe head, with silicon lens and covers.

The material that covers the transducer and acts as focusing lens for the ultrasound beam is typically a silicon rubber convex lens, which must be considered as an hyperelastic material.

3.2. Transducer characterization through measurements

The transducer's fundamental performances can be evaluated by measurement of electrical impedance and far field sound pressure level; whose quality and reliability play an important role in the comparison with simulation results. For this purpose it is very important to minimize all possible parasitic effects which could result in a misleading measurement. For further details, check our previous work [1].
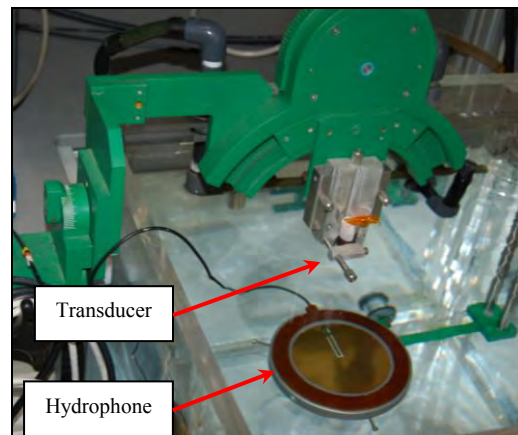

Figure 5: Pressure measurement setup.

As regard measurement instrumentation, we have used: Hewlett Packard 4195A Network Analyzer, Agilent 33250A waveform generator, LeCroy LT342 oscilloscope and an high bandwidth membrane hydrophone placed in a thermostated, demineralized water tank (see Figure 5).

4. KLM model

In order to determine the starting values of matching layers" thickness and acoustic impedance, we considered the single piezoelectric element chosen for the linear array probe (with dimensions: 6×0.21 mm, 0.3 mm thick) and applied the microwave multi–sections transmission line theory [7], neglecting the presence of the silicon rubber lens[1].

Setting the thickness of the n–matching layer to $\lambda_n/4$ where λ_n is the central wavelength calculated in the n–matching layer, one can choose at least two different approaches:

1) Binomial or maximally flat response and
2) Chebyschev or equi–ripple response.

For a given number of sections, the first approach leads to a frequency response that is as flat as possible near the design frequency (i.e. 5 MHz in our case). The second approach optimizes bandwidth at the expense of the passband ripple.

Taking into account that a short pulse–response with minimum ringing could be obtained by using the Binomial approach [10], we chose this one. In this case the matching layers" acoustic impedance are defined by (approximate expression, [7]):

$$\ln\left(\frac{Z_{n+1}}{Z_n}\right) = 2^{-N}\frac{N!}{(N-n)!n!}\ln\left(\frac{Z_f}{Z_0}\right) \quad (7)$$

where Z_n is the n–matching layer" impedance, N is the total number of matching layer (4 in the present case), Z_0 is the acoustic impedance of the piezoelectric material (22.2 MRayls, see §3.1) and Z_f is the acoustical impedance of the biological medium (~1.5 MRayls). A solution for the binomial response design is reported in Table 1.

Using the matching layer parameter values reported in Table 1, the simulated sound pressure level leaving the single piezoelectric element $p_0 = p|_{(x,0)}$ is reported in Figure 6 (with 1 V driving voltage).

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[1] The silicon rubber lens affects the electro-acoustical response of the transducer, due to its high attenuation coefficient, but doesn"t affect the above matching layers" design procedure, since the values of acoustic impedance for the silicon rubber and for the biological medium final load are very close.

Matching layer:	Sound speed [m/s]	Thickness [wavelength]	Thickness [μm]	Z_n [MRayls]
1st layer	1500	$\lambda_1/4$	75 μm	18.8
2nd layer	1700	$\lambda_2/4$	85 μm	9.6
3rd layer	2700	$\lambda_3/4$	135 μm	3.5
4th layer	1800	$\lambda_4/4$	90 μm	1.8

Table 1: matching layers thickness and acoustic impedance values, optimized for maximally flat response around 5 MHz.
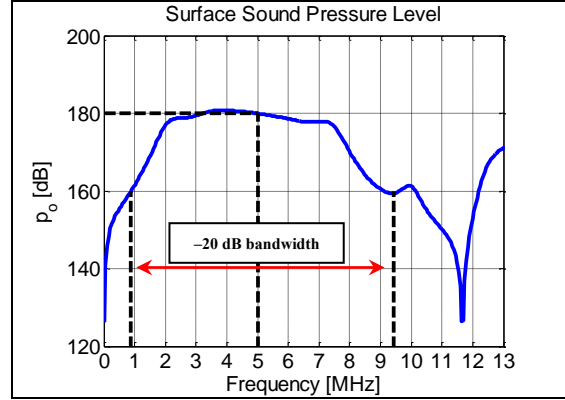

Figure 6: KLM maximally flat sound pressure level from single piezoelectric element (1 V driving voltage).

From Figure 6 it is clear that the sound pressure level is maximum around the design center frequency (5 MHz) and practically flat within a large neighborhood, but falls off very quickly at higher frequency, so that the desired performances are not satisfied, in fact the –20 dB bandwidth is from about 880 kHz up to 9.4 MHz.

Matching layer:	Sound speed [m/s]	Thickness [wavelength]	Thickness [μm]	Z_n [MRayls]
1st layer	1500	$\lambda_1/5$	60 μm	8.5
2nd layer	1700	$\lambda_2/5.67$	60 μm	6.0
3rd layer	2700	$\lambda_3/9$	60 μm	3.0
4th layer	1800	$\lambda_4/6$	60 μm	2.0

Table 2: matching layers thickness and acoustic impedance values, optimized for input specifications fulfillment.

In order to satisfy the input specifications in term of bandwidth, without lowering the

maximum sound pressure level around the center frequency, we decided to vary the values of both thickness and acoustical impedance of the matching layers: the final KLM optimized values are reported in Table 2.

The corresponding sound pressure level leaving the single piezoelectric element p_0 is reported on Figure 7 (for 1 V driving voltage).
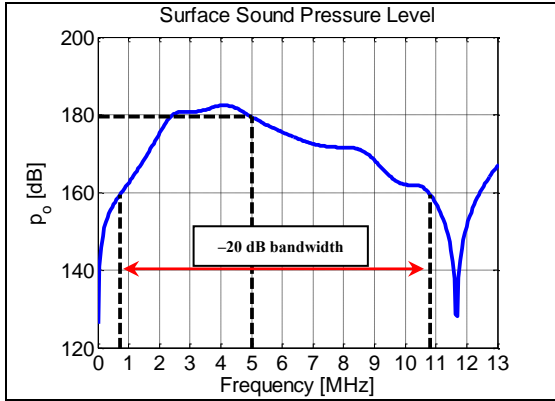

Figure 7: KLM sound pressure level from single piezoelectric element, optimized for 2 MHz – 11 MHz bandwidth (1 V driving voltage).

The result in Figure 7 shows that the sound pressure level p_0 has roughly the same values (180 dB) of the KLM maximally flat design at the center frequency, but falls off more slowly at high frequency (with respect to the previous case, see Figure 6) and the –20 dB bandwidth starts from about 730 kHz and ends at about 11 MHz. In this case, the matching layers'' stack fulfills the prescribed performances.

5. Use of COMSOL Multiphysics

5.1. Piezoelectric characterization
The piezoelectric material requires a different description in COMSOL (3 matrices need to be determined, see §2.2) with respect to KLM, where only material parameters for the polarization direction are needed, so a preliminary FEM validation and optimization procedure for the piezoelectric material alone is needed. Therefore, since the electrical impedance measurement was not possible for the single array element alone (i.e. not bonded on backing), the first analysis consists in the impedance frequency response of the whole plate (6×36 mm, 0.3 mm thick), compared with

measurements. This last COMSOL model is not reported here.

The optimization procedure was performed as described in §2.3 and the measured (solid line) and simulation results (dotted line) are reported in Figure 8.
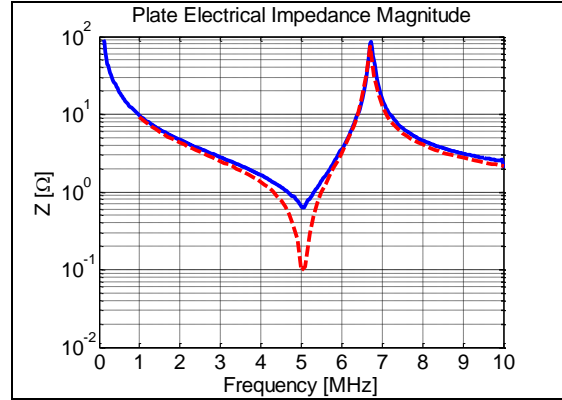

Figure 8: Piezocomposite plate (6×36 mm, 0.3 mm thick) electrical impedance magnitude: measured (solid), FEM (dotted).

The final agreement between measurement and simulation results is good over the whole frequency range. In particular, resonance and anti–resonance frequency fit error is less that 3%.

The first thickness vibration mode is clearly placed at 5 MHz and no low frequency transverse vibration modes are evident. As regard the optimization procedure details, these are the same as reported in our previous work [1]. The parameter determined for the piezomaterial are then used for the complete transducer FEM.

5.2. Building the finite element model
The FEM for the transducer array was built using the COMSOL acoustics module, coupling the piezoelectric and pressure acoustics applications, in 2D space dimension. The optimization module was finally used to fit the model simulation to the measurement results starting from the values developed using KLM model.

Mesh on all domains was chosen as free tetrahedral. In order to consider a quasi–static approximation for each elementary triangular element, the segment length should be shorter

than approximately $\lambda_{min}/5$ [2], where λ_{min} is the minimum wavelength of the system analysis. This allows a good compromise between computational time and accuracy results.

In order to reduce the complete FEM node number, the acoustic domain (i.e. water, equivalent to biological medium) was reduced to a small region surrounded by Perfectly Matched Layer (PML), which simulate the zero reflection condition. Then far field sound pressure level was calculated, as previously discussed. Moreover, only a set of central elements was modeled and a symmetry condition was used (Figure 9 shows the 2D FEM).
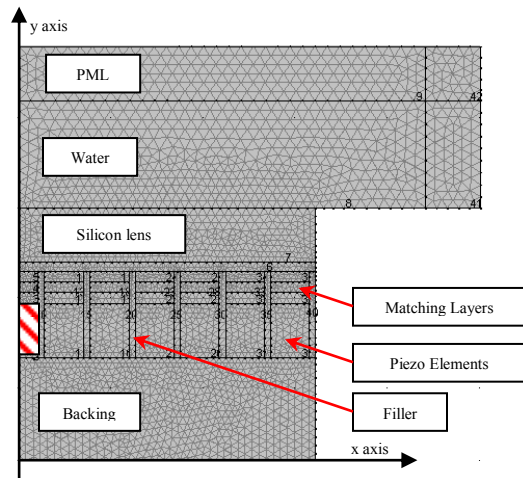

Figure 9: Transducer COMSOL 2D FEM. Red striped block: active piezoelectric element.

5.3. Complete probe characterization

After the piezocomposite plate alone was studied, the complete probe was implemented in COMSOL, including the four matching layers developed with KLM model and the silicon rubber acoustic lens, on top of the matching layers" stack (see §5.2). Silicon rubber should be considered a hyperelastic material, so it is described by a couple of *Lamè* constants instead of *Young* modulus and *Poisson* coefficient.

With the complete FEM, the far field sound pressure level, p_{far}, generated by a single element of the array was studied (Red striped block in Figure 9). The frequency response

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[2] If the hardware constraints (memory, CPU speed) allow it, an element size of $\lambda/10$ is preferred.

analysis was run using the measured driving voltage (approximately 20 V [3] with a variation of less than 5% in the whole frequency range) as electrical boundary condition for the array element. The far field sound pressure level was calculated as explained in §2.2 at a distance of 60 mm from the transducer surface. Figure 10 shows the comparison (in dB) between measured (solid line) and FEM results (dotted line). The agreement between measurement and simulation results can be considered very good, with less than 3 dB of amplitude differences all along the 2 MHz – 11 MHz operating range.
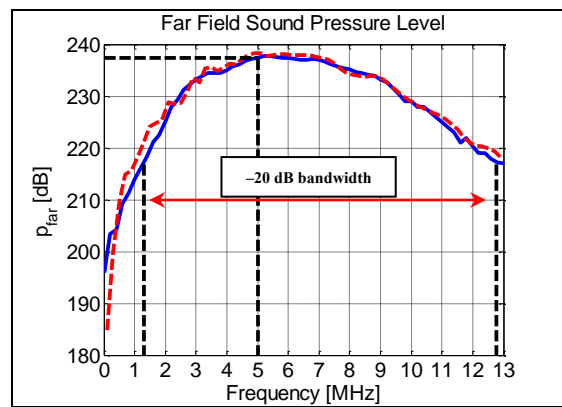

Figure 10: Far field sound pressure level (dB) at a distance of 60 mm from the transducer surface: measured (solid) and simulated (dotted).

5.4. Directivity simulations

A very important task in the design process of an ultrasound imaging probe is the optimization of the beam steering capabilities of the array. In many operation modes (in particular Color Flow Mode (CFM), Doppler imaging), it is needed to "*steer*" the ultrasound beam propagating into the human body, introducing phase differences in the driving signal of the array set of active elements. On the other hand, the array will be able to keep a good beam shape only within a range of angles from axis, never exceeding 30 degrees. Such "*steering*" capability of the array is determined by the radiation lobe amplitude of the single element, as this determines how much interaction between emission from adjacent elements is possible.

Once the model for the complete probe is optimized, FEM becomes an extremely easy and

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[3] 26 dB more with respect the KLM model.

efficient tool to determine the radiation lobe of the single element and the beam steering capabilities of the array. In the following, it will be clear how it is possible to relate these important probe performances to the mechanical properties of the materials.

As an interesting example of application we considered to change the silicon rubber lens to a material with a lower hyperelasticity, corresponding to a lower value for the *Lamè* λ parameter. The two radiation lobes are reported in the following Figure 11 (for 5 MHz excitation frequency):
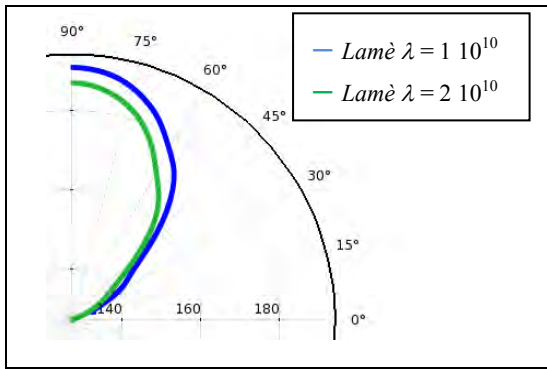

Figure 11: Sound pressure level (dB) emitted from single element at the acoustic interface, with 5 MHz excitation frequency.

As can be clearly noticed, the decrease of probe lens hyperelasticity leads to a larger radiation lobe, which is highly desired for beam steering capability, as it¨s confirmed from the following final simulation.

6. Beam steering

The actual operating mode for an imaging probe consists in the simultaneous excitation of a set of elements, in order to form the ultrasound beam in the biological medium.

If proper delay values are introduced in the driving voltage for a set of array elements (i.e. 12 active elements in our case), the resulting acoustic beam will be steered by a certain angle. From geometrical calculations, the following delay function $\Delta T(x_n)$ can be deduced:

$$\Delta T(x_n) = \frac{2\pi f}{v}\left(y_c - \sqrt{F^2 - (x_n - x_c)^2}\right) \quad (8)$$

where v is the ultrasound propagation velocity in the medium, f is the excitation frequency, x_n is the position of the n–element of the array (with

$n=0$ in the center) and (x_c, y_c) are the coordinates of the desired beam focus point, with focus distance F and steering angle θ, given by:

$$\begin{cases} x_c = F\sin\theta \\ y_c = F\cos\theta \end{cases} \quad (9)$$

The following simulation results (Figure 12) obtained with $\theta = 25°$ steered beam, $F = 20$ mm, $f = 5$ MHz, 12 active elements, confirmed that an acoustic lens with lower hyperelasticity would improve the steering capability, as a higher acoustic power would be available for large steering angles:
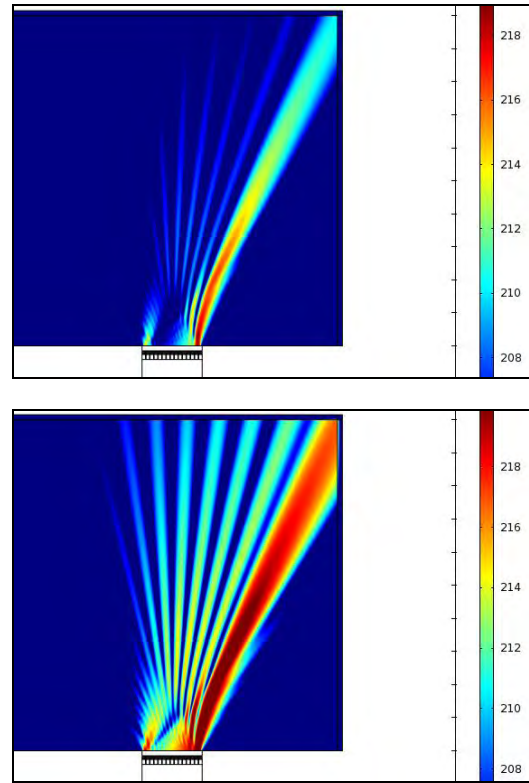

Figure 12: Sound pressure level map. $\theta = 25°$ steered beam, $F = 20$ mm, $f = 5$ MHz, 12 active elements. Upper: Standard silicon lens, *Lamè* $\lambda = 2\ 10^{10}$. Lower: Lower hyperelasticity lens, *Lamè* $\lambda = 1\ 10^{10}$.

7. Conclusions

Both a mono dimensional electro–acoustical KLM and a FEM model have been used to design a 5 MHz linear imaging probe. KLM has been used to find the starting values for the principal parameters involved in the model (i.e.

matching layer's stack) while FEM has been used for the complete modeling of the probe.

Final results for the far field sound pressure level show a good agreement between measured and simulated transducer performances, with less than 3 dB of amplitude differences all along the bandwidth operating range.

Directivity and *"beam steering"* simulations prove that FEM can greatly help in understanding how probe performances could be improved. Indeed it was possible to relate the mechanical properties of the acoustical lens of the probe to its steering capability.

8. References

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